ELEMENTARY
CORE Academy
UTAH STATE OFFICE OF EDUCATION & UTAH STATE UNIVERSITY

Participant Handbook

ELEMENTARY CORE ACADEMY • ELEMENTARY CORE ACADEMY •

5TH GRADE

2007

UTAH STATE OFFICE OF EDUCATION

ELEMENTARY CORE ACADEMY

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Organizations:

Utah State Office of Education (USOE)
Utah State University (USU)
State Science Education Coordination Committee (SSECC)
State Mathematics Education Coordination Committee (SMECC)
Special Education Services Unit (USOE)

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UTAH STATE OFFICE OF EDUCATION

Leadership…Service…Accountability

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Dear CORE Academy Teachers:

Thank you for your investment in children and in building your own expertise as you participate in the Elementary CORE Academy. I hope your involvement helps you to sustain a laser-like focus on student achievement.

Teachers in Utah are superb. By participating in the Academy, you join a host of teachers throughout the state who understand that teaching targeted on the core curricula, across a spectrum of subjects, will produce results of excellence. The research is quite clear–the closer the match of explicit instruction to core standards, the better the outcome on core assessments.

I personally appreciate your excellence and your desire to create wonderful classrooms of learning for students. Thank you for your dedication. I feel honored to associate with you and pledge my support to lead education in ways that benefit all of our children.

Sincerely,

Patti Harrington

Patti Harrington, Ed.D.
State Superintendent of Public Instruction

Funding Sources

Appreciation is expressed for the tremendous educational input and monetary commitment of several organizations for the successful delivery of the Elementary CORE Academy. This year's Elementary CORE Academy was developed and funded through a variety of sources. The Utah State Office of Education (USOE), in collaboration with Utah State University (USU) and local school districts of Utah, have supported kindergarten through sixth grade teachers with professional development experiences that will enhance the educational experience for Utah children.

Major funding for the Academy comes from the following sources:

Federal/State Funds:

Utah State Office of Education
- Staff Development Funds
- Special Education Services Unit

ESEA Title II

Utah Math Science Partnership

District Funds:

Various sources including Quality Teacher Block, Federal ESEA Title II, and District Professional Development Funds

School Funds:

Trust land, ESEA Title II, and other school funds
Utah State Office of Education Special Education Services

The state and district funds are allocations from the state legislature. ESEA is part of the "No Child Left Behind" funding that comes to Utah.

Additionally, numerous school districts, individual schools, and principals in Utah have sponsored teachers to attend the Academy. Other educational groups have assisted in the development and delivery of resources in the Academy.

Most important is the thousands of teachers who take time from their summer to attend these professional development workshops. It is these teachers who make this program possible.

Goals of the Elementary CORE Academy

Overall

The purpose of the Elementary CORE Academy is to create high quality teacher instruction and improve student achievement through the delivery of professional development opportunities and experiences for teachers across Utah.

The Academy will provide elementary teachers in Utah with:

1. Models of exemplary and innovative instructional strategies, tools, and resources to meet the Core Curriculum standards, objectives, and indicators.
2. Practical models and diverse methods of meeting the learning needs of all children, with instruction implementation aligned to the Core Curriculum.
3. Meaningful opportunities for collaboration, self-reflection, and peer discussion specific to innovative and effective instructional techniques, materials, teaching strategies, and professional practices in order to improve classroom instruction.

Learning a limited set of facts will no longer prepare a student for real experiences encountered in today's world. It is imperative that educators have continued opportunities to obtain instructional skills and strategies that provide methods of meeting the needs of all students. Participants of the Academy experience will be better equipped to meet the challenges faced in today's classrooms.

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Math
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Utah Elementary Math Core Curriculum

Introduction

Most children enter school confident in their own abilities; they are curious and eager to learn more. They make sense of the world by reasoning and problem solving. Young students are building beliefs about what mathematics is, about what it means to know and do mathematics, and about themselves as mathematical learners. Students use mathematical tools, such as manipulative materials and technology, to develop conceptual understanding and solve problems as they do mathematics. Students, as mathematicians, learn best through participatory experiences throughout the instruction of the mathematics curriculum.

Recognizing that no term captures completely all aspects of expertise, competence, knowledge, and facility in mathematics, the term *mathematical proficiency* has been chosen to capture what it means to learn mathematics successfully. Mathematical proficiency has five strands: computing (carrying out mathematical procedures flexibly, accurately, efficiently, and appropriately), understanding (comprehending mathematical concepts, operations, and relations), applying (ability to formulate, represent, and solve mathematical problems), reasoning (logically explaining and justifying a solution to a problem), and engaging (seeing mathematics as sensible, useful, and doable, and being able to do the work) (NRC, 2001).

The most important observation about the five strands of mathematical proficiency is that they are interwoven and interdependent. This observation has implications for how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve that goal. At any given moment during a mathematics lesson or unit, one or two strands might be emphasized. But all the strands must eventually be addressed so that the links among them are strengthened. The integrated and balanced development of all five strands of mathematical proficiency should guide the teaching and learning of school mathematics. Instruction should not be based on the extreme positions that students learn solely by internalizing what a teacher or book says, or solely by inventing mathematics on their own.

The Elementary Mathematics Core describes what students should know and be able to do at the end of each of the K-6 grade levels. It was developed and revised by a community of Utah mathematics

teachers, mathematicians, university mathematics educators, and State Office of Education specialists. It was critiqued by an advisory committee representing a wide variety of people from the community, as well as an external review committee. The Core reflects the current philosophy of mathematics education that is expressed in national documents developed by the National Council of Teachers of Mathematics, the American Association for the Advancement of Science, and the National Research Council. This Mathematics Core has the endorsement of the Utah Council of Teachers of Mathematics. The Core reflects high standards of achievement in mathematics for all students.

Organization of the Elementary Mathematics Core

The Core is designed to help teachers organize and deliver instruction.

- Each grade level begins with a brief description of areas of instructional emphasis which can serve as organizing structures for curriculum design and instruction.
- The INTENDED LEARNING OUTCOMES (ILOs) describe the skills and attitudes students should acquire as a result of successful mathematics instruction. They are found at the beginning of each grade level and are an integral part of the Core.
- A STANDARD is a broad statement of what students are expected to understand. Several Objectives are listed under each Standard.
- An OBJECTIVE is a more focused description of what students need to know and be able to do at the completion of instruction. If students have mastered the Objectives associated with a given Standard, they have mastered that Standard at that grade level. Several Indicators are described for each Objective.
- INDICATORS are observable or measurable student actions that enable students to master an Objective. Indicators can help guide classroom instruction.
- MATHEMATICAL LANGUAGE AND SYMBOLS STUDENTS SHOULD USE includes language and symbols students should use in oral and written language.
- EXPLORATORY CONCEPTS AND SKILLS are included to establish connections with learning in subsequent grade levels. They are not intended to be assessed at the grade level indicated.

Guidelines Used in Developing the Elementary Mathematics Core

The Core is:

Consistent With the Nature of Learning

In the early grades, children are forming attitudes and habits for learning. It is important that instruction maximizes students' potential and gives them understanding of the intertwined nature of learning. The main intent of mathematics instruction is for students to value and use mathematics as a process to understand the world. The Core is designed to produce an integrated set of Intended Learning Outcomes for students.

Coherent

The Core has been designed so that, wherever possible, the ideas taught within a particular grade level have a logical and natural connection with each other and with those of earlier grades. Efforts have also been made to select topics and skills that integrate well with one another and with other subject areas appropriate to grade level. In addition, there is an upward articulation of mathematical concepts and skills. This spiraling is intended to prepare students to understand and use more complex mathematical concepts and skills as they advance through the learning process.

Developmentally Appropriate

The Core takes into account the psychological and social readiness of students. It builds from concrete experiences to more abstract understandings. The Core focuses on providing experiences with concepts that students can explore and understand in depth to build the foundation for future mathematical learning experiences.

Reflective of Successful Teaching Practices

Learning through play, movement, and adventure is critical to the early development of the mind and body. The Core emphasizes student exploration. The Core is designed to encourage a variety of interactive learning opportunities. Instruction should include recognition of the role of mathematics in the classroom, school, and community.

Comprehensive

By emphasizing depth rather than breadth, the Elementary Mathematics Core seeks to empower students by providing a comprehensive background in mathematics. Teachers are expected to teach all the

The Core is:

- Coherent
- Developmentally Appropriate
- Encourages Good Teaching Practices
- Comprehensive
- Feasible
- Useful and Relevant
- Encourages Good Assessment Practices

standards and objectives specified in the Core for their grade level, but may add related concepts and skills.

Feasible

Teachers and others who are familiar with Utah students, classrooms, teachers, and schools have designed the Core. It can be taught with easily obtained resources and materials. A handbook is also available for teachers and has sample lessons on each topic for each grade level. The handbook is a document that will grow as teachers add exemplary lessons aligned with the new Core.

Useful and Relevant

This curriculum relates directly to student needs and interests. The relevance of mathematics to other endeavors enables students to transfer skills gained from mathematics instruction into their other school subjects and into their lives outside the classroom.

Reliant Upon Effective Assessment Practices

Student achievement of the standards and objectives in this Core is best assessed using a variety of assessment instruments. Performance tests are particularly appropriate to evaluate student mastery of mathematical processes and problem-solving skills. Teachers should use a variety of classroom assessment approaches in conjunction with standard assessment instruments to inform instruction. Sample test items, keyed to each Core Standard, may be located on the "Utah Mathematics Home Page" at http://www.usoe.k12.ut.us/curr/math. Observation of students engaged in instructional activities is highly recommended as a way to assess students' skills as well as attitudes toward learning. The nature of the questions posed by students provides important evidence of their understanding of mathematics.

Based Upon the National Council of Teachers of Mathematics Curriculum Focal Points

In 2006, the National Council of Teachers of Mathematics (NCTM) published *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM, 2006). This document is available online at http://www.nctm.org/focalpoints. This document describes three focal points for each grade level. NCTM's focal points are areas of emphasis recommended for the curriculum of each grade level. The focal points within a grade are ***not the entire curriculum*** for that particular grade; however, Utah's Core Curriculum was designed to include these areas of focus.

Intended Learning Outcomes for Third through Sixth Grade Mathematics

The main intent of mathematics instruction is for students to value and use mathematics and reasoning skills to investigate and understand the world.

The Intended Learning Outcomes (ILOs) describe the skills and attitudes students should acquire as a result of successful mathematics instruction. They are an essential part of the Mathematics Core Curriculum and provide teachers with a standard for student learning in mathematics.

ILOs for mathematics:

1. **Develop a positive learning attitude toward mathematics.**
2. **Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.**
3. **Reason logically, using inductive and deductive strategies and justify conclusions.**
4. **Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.**
5. **Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.**
6. **Represent mathematical ideas in a variety of ways.**

Significant mathematics understanding occurs when teachers incorporate ILOs in planning mathematics instruction. The following are ideas to consider when planning instruction for students to acquire the ILOs:

1. **Develop a positive learning attitude toward mathematics.**

   When students are confident in their mathematical abilities, they demonstrate persistence in completing tasks. They pose mathematical questions about objects, events, and processes while displaying a sense of curiosity about numbers and patterns. It is important to build on students' innate problem-solving inclinations and to preserve and encourage a disposition that values mathematics.

2. **Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.**

   Problem solving is the cornerstone of mathematics. Mathematical knowledge is generated through problem solving

as students explore mathematics. To become effective problem solvers, students need many opportunities to formulate questions and model problem situations in a variety of ways. They should generalize mathematical relationships and solve problems in both mathematical and everyday contexts.

3. **Reason logically, using inductive and deductive strategies and justify conclusions.**

   Mathematical reasoning develops in classrooms where students are encouraged to put forth their own ideas for examination. Students develop their reasoning skills by making and testing mathematical conjectures, drawing logical conclusions, and justifying their thinking in developmentally appropriate ways. Students use models, known facts, and relationships to explain reasoning. As they advance through the grades, students' arguments become more sophisticated.

4. **Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.**

   The ability to express mathematical ideas coherently to peers, teachers, and others through oral and written language is an important skill in mathematics. Students develop this skill and deepen their understanding of mathematics when they use accurate mathematical language to talk and write about what they are doing. When students talk and write about mathematics, they clarify their ideas and learn how to make convincing arguments and represent mathematical ideas verbally, pictorially, and symbolically.

5. **Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.**

   Students develop a perspective of the mathematics field as an integrated whole by understanding connections within mathematics. Students should be encouraged to explore the connections that exist with other disciplines and between mathematics and their own experiences.

6. **Represent mathematical ideas in a variety of ways.**

   Mathematics involves using various types of representations including concrete, pictorial, and symbolic models. In particular, identifying and locating numbers on the number line has a central role in uniting all numbers to promote understanding of equivalent representations and ordering. Students also use a variety of mathematical representations to expand their capacity to think logically about mathematics.

Fifth Grade Mathematics Standards

By the end of grade five, students increase their facility with the four basic arithmetic operations applied to whole numbers, fractions, and decimals. They locate integers on a number line and ordered pairs of integers on the coordinate plane. They determine rules for numerical patterns, work with expressions including order of operations, and solve single-operation equations involving a single variable. They classify angles, triangles, and quadrilaterals, and analyze relationships among lines, triangles and quadrilaterals. They recognize and determine surface area and volume of three-dimensional shapes, including right prisms. Students understand the concepts of mean, median, mode, and range of data sets and can calculate them. They use line plots, bar graphs, and line graphs to record and analyze data.

Standard I: Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Standard I: Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 1: Represent whole numbers and decimals from thousandths to one billion, fractions, percents, and integers.

a. Read and write numbers in standard and expanded form.

b. Demonstrate multiple ways to represent whole numbers, decimals, fractions, percents, and integers using models and symbolic representations (e.g., 108 = 2 x 50 + 8; 108 = 10^2 + 8; 90% = 90 out of 100 squares on a hundred chart).

c. Identify, read, and locate fractions, mixed numbers, decimals, and integers on the number line.

d. Represent repeated factors using exponents.

e. Describe situations where integers could be used in the students' environment.

Objective 2: Explain relationships and equivalencies among integers, fractions, decimals, and percents.

a. Compare fractions by finding a common denominator.

b. Order integers, fractions (including mixed numbers), and decimals using a variety of methods, including the number line.

c. Rewrite mixed numbers and improper fractions from one form to the other and represent each using regions, sets of objects, or line segments.

d. Represent commonly used fractions as decimals and percents in a variety of ways (e.g., models, fraction strips, pictures, calculators, algorithms).

e. Model and calculate equivalent forms of a fraction (including simplest form).

f. Rename whole numbers as fractions with different denominators (e.g., 5 = 5/1, 3 = 6/2, 1 = 7/7).

Objective 3: Use number theory concepts to develop and use divisibility tests; classify whole numbers to 50 as prime, composite, or neither; and find common multiples and factors.

a. Identify patterns with skip counting and multiples to develop and use divisibility tests for determining whether a whole number is divisible by 2, 3, 5, 6, 9, and 10.

b. Use strategies for classifying whole numbers to 50 as prime, composite, or neither.

c. Rewrite a composite number between 2 and 50 as a product of only prime numbers.

d. Find common multiples and factors and apply to adding and subtracting fractions.

Objective 4: Model and illustrate meanings of multiplication and division.

a. Represent division-with-remainder using whole numbers, decimals, or fractions.

b. Describe the effect of place value when multiplying and dividing whole numbers and decimals by 10, 100, and 1,000.

c. Model multiplication of fractions and decimals (e.g., tenths multiplied by tenths, a whole number multiplied by tenths, or a whole number with tenths multiplied by tenths) in a variety of ways (e.g., manipulatives, number line and area models, patterns).

Objective 5: Solve problems involving one or two operations.

a. Determine when it is appropriate to use estimation, mental math strategies, paper and pencil, and algorithms.

b. Make reasonable estimations of fraction and decimal sums, differences, and products, including knowing whether results obtained using a calculator are reasonable.

c. Write number sentences that can be used to solve a two-step problem.

d. Interpret division-with-remainder problems as they apply to the environment (e.g., If there are 53 people, how many vans are needed if each van holds 8 people?).

Objective 6: Demonstrate proficiency with multiplication and division of whole numbers and compute problems involving addition, subtraction, and multiplication of decimals and fractions.

a. Multiply multi-digit whole numbers by a two-digit whole number with fluency, using efficient procedures.

b. Divide multi-digit dividends by a one-digit divisor with fluency, using efficient procedures.

c. Add and subtract decimals with fluency, using efficient procedures.

d. Add and subtract fractions with fluency.

e. Multiply fractions.

Mathematical language and symbols students should use

prime, composite, exponent, fractions, numerator, denominator, common denominator, common factor, common multiple, decimals, percents, divisible, divisibility, equivalent fractions, integer, dividend, quotient, divisor, factor, order of operations, simplest terms, various symbols for multiplication and division, mixed numeral, improper fraction

Exploratory Concepts and Skills

- Extend classification of whole numbers from 0-100 as prime, composite, or neither.
- Apply rules of divisibility.
- Explore adding and subtracting integers.
- Divide multi-digit dividends by a two-digit divisor.

Standard II: Students will use patterns and relations to represent and analyze mathematical problems and number relationships using algebraic symbols.

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Objective 1: Identify, analyze and determine a rule for predicting and extending numerical patterns involving operations whole numbers, decimals, and fractions.

a. Analyze and make predictions about numeric patterns, including decimals and fractions.

b. Determine a rule for the pattern using organized lists, tables, objects, and variables.

Objective 2: Use algebraic expressions, inequalities, or equations to represent and solve simple real-world problems. –

a. Use properties and the order of operations involving addition, subtraction, multiplication, division, and the use of parentheses to compute with whole numbers, decimals, and fractions.

b. Use patterns, models, and relationships as contexts for writing and solving simple equations and inequalities with whole number solutions (e.g., 6x = 54; x + 3 = 7).

Mathematical language and symbols students should use

variety of symbols for multiplication and division such as x, i, and * as symbols for multiplication and ÷, e, and a fraction bar (/ or —) as division symbols; variable, order of operations, parentheses, inequality, expression, equation, associative property, commutative property, distributive property

Exploratory Concepts and Skills

- Extend classification of whole numbers from 0-100 as prime, composite, or Solve multi-step equations.
- Construct and analyze tables involving equivalent ratios.

Standard III: Students will use spatial reasoning to recognize, describe, and analyze geometric shapes and principles.

Objective 1: Describe relationships between two- and three-dimensional shapes and analyze attributes and properties of geometric shapes.

a. Draw, label, and describe line segments, rays, lines, parallel lines, and perpendicular lines.

b. Draw, label, and define an angle as two rays sharing a common endpoint (vertex).

c. Classify triangles and quadrilaterals and analyze the relationships among the shapes in each classification (e.g., a square is a rectangle).

d. Relate pyramids and right prisms to the two-dimensional shapes (nets) from which they were created.

e. Identify properties and attributes of solids (i.e., right prisms, pyramids, cylinders, cones) and describe them by the number of edges, faces, and vertices as well as the types of faces.

Objective 2: Specify locations in a coordinate plane.

a. Locate points defined by ordered pairs of integers.

b. Write an ordered pair for a point in a coordinate plane with integer coordinates.

c. Specify possible paths between locations on a coordinate plane and compare distances of the various paths.

Mathematical language and symbols students should use

perpendicular and parallel lines, rays, angles (acute, obtuse, right, straight), triangles (equilateral, isosceles, scalene, right, acute, obtuse), vertex, vertices, edge, face, corresponding angles, similar, polygon, pyramid, right prism

Exploratory Concepts and Skills

- Compare corresponding angles of two triangles and determine whether the triangles are similar.
- Rotate a shape around a fixed point and identify the location of the new vertices.
- Translate a polygon either horizontally or vertically on a coordinate grid and identify the location of the new vertices.
- Reflect a shape across either the x- or y-axis and identify the location of the new vertices.

Standard IV: Students will determine area of polygons and surface area and volume of three-dimensional shapes.

Standard IV: Students will determine area of polygons and surface area and volume of three-dimensional shapes.

Objective 1: Determine the area of polygons and apply to real-world problems.

a. Determine the area of a trapezoid by the composition and decomposition of rectangles, triangles, and parallelograms.

b. Determine the area of irregular and regular polygons by the composition and decomposition of rectangles, triangles, and parallelograms.

c. Compare areas of polygons using different units of measure within the same measurement system (e.g., square feet, square yards).

Objective 2: Recognize, describe, and determine surface area and volume of three-dimensional shapes.

a. Quantify volume by finding the total number of same-sized units of volume needed to fill the space without gaps or overlaps.

b. Recognize that a cube having a 1 unit edge is the standard unit for measuring volume expressed as a cubic unit.

c. Derive and use the formula to determine the volume of a right prism with a triangular or rectangular base.

d. Relate the formulas for the areas of triangles, rectangles, or parallelograms to the surface area of a right prism.

e. Derive and use the formula to determine the surface area of a right prism and express surface area in square units.

Mathematical language and symbols students should use
area, volume, surface area, volume, right prism

Exploratory Concepts and Skills
- Investigate pi as the ratio of the circumference to the diameter of a circle.
- Determine the volume of a right prism with various bases.

Standard V: Students will construct, analyze, and construct reasonable conclusions from data and apply basic concepts of probability.

Objective 1: Formulate and answer questions using statistical methods to compare data, and propose and justify inferences based on data.

a. Construct, analyze, and display data using an appropriate format (e.g., line plots, bar graphs, line graphs).

b. Recognize the differences in representing categorical and numerical data.

c. Identify minimum and maximum values for a set of data.

d. Identify and calculate the mean, median, mode, and range.

Objective 2: Apply basic concepts of probability.

a. Describe the results of experiments involving random outcomes using a variety of notations (e.g., 4 out of 9, 4/9).

b. Recognize that probability is always a value between 0 and 1 (inclusively).

c. Express the likelihood of an outcome in a simple experiment as a value between 0 and 1 (inclusively).

Mathematical language and symbols students should use

data, minimum values, maximum values, mean, median, mode, average, range

Exploratory Concepts and Skills

- Explore the differences in representing categorical and numerical data.

Facilitated

Activities

New Math Core Curriculum
Elementary CORE Academy 2007

Since the 2003 adoption of Utah's Elementary Mathematics Core Curriculum, ideas such as coherence, focus, high expectations, computational fluency, representation, and important mathematics have become regular elements in discussions about improving school mathematics. As the next step in devising resources to support the development of a coherent curriculum, the National Council of Teachers of Mathematics (NCTM) released *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence.*

With NCTM's release of the Curriculum Focal Points and discussion regarding high expectations, it became important for Utah to revise the Elementary Mathematics Core Curriculum. The placement of concepts within the Curriculum Focal Points guided the placement of concepts within Utah's Core.

The Core has also been designed so that, wherever possible, the ideas taught within a particular grade level have a logical and natural connection with each other and with those of earlier grades. Efforts have also been made to select topics and skills that integrate well with one another and with other subject areas appropriate to grade level. In addition, there is an upward articulation of mathematical concepts and skills. This spiraling is intended to prepare students to understand and use more complex mathematical concepts and skills as they advance through the learning process.

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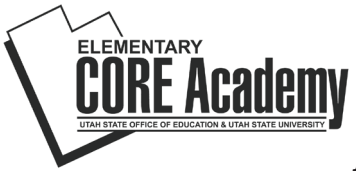
The Elementary Mathematics Core describes what students should know and be able to do at the end of each of the K-6 grade levels. It was developed and revised by a community of Utah mathematics teachers, mathematicians, university mathematics educators, and State Office of Education specialists. It was critiqued by an advisory committee representing a wide variety of people from the community, as well as an external review committee. The Core reflects the current philosophy of mathematics education that is expressed in national documents developed by the National Council of Teachers of Mathematics, the American Association for the Advancement of Science, and the National Research Council. This Mathematics Core has the endorsement of the Utah Council of Teachers of Mathematics. The Core reflects high standards of achievement in mathematics for all students.



E-D-P Model
Elementary CORE Academy 2007

Each day good educators observe and interact with students to determine what course of action should be taken to achieve the best educational results for each learner. These observations, in many instances, are made with limited formal data. The E-D-P Model assists educators in the collection and use of information justifying implementation of practices. Many educators struggle with the ability to articulate and align teaching actions with student learning needs. The E-D-P Model is a method of aiding this articulation.

When assessing, it is important to know that correct answers do not necessarily mean students understand a concept. Conversely, incorrect responses may not indicate that a student hasn't learned a concept. It is important for educators to look for hidden understandings and possible misconceptions. Ongoing assessments, observations, and interviews may be necessary. When using this process, instructors should select assignments/tasks where students have opportunities to explain their understanding. Developing a tool to aid teachers in the collection of information and to assist them in determining student understanding has been the driving force in creating the E-D-P Model.

Our discussion begins with a description of the E-D-P Model. This model is based on a medical metaphor of **E**valuation-**D**iagnosis-**P**rescription (E-D-P). It is important to understand the difference between three main types of assessment: diagnostic (usually occurring prior to instruction), formative (concurrently occurs with instruction), and summative (occurs at the conclusion of an instructional period). The E-D-P Model targets diagnostic and formative assessments. By conducting ongoing assessments and using this formative information, educators can effectively impact student learning and plan instruction to meet individual learning needs (McNamee & Chen, 2005).

Evaluation

In classrooms across the country one may observe teachers interacting with students in a variety of ways. The **E**valuation portion of the E-D-P Model provides teachers with a way to identify student learning as it relates to the standard and objective of instruction. As a teacher sees a particular student response she is able to identify understandings and misunderstandings.

EXAMPLE: Marcia responded with the answer of 12 when she was asked to add 14 and 8. Using Marcia's work, an instructor sees that Marcia needs instruction on renaming. Other conclusions for the same response may also be apparent. The **E**valuation phase can then transition to the **D**iagnosis.

Diagnosis

As the student response is investigated the instructor may need to ask questions or inquire regarding the reasoning used to formulate the response. This is similar to a physician, where if a pain in the abdomen is described, the doctor poses questions to the patient or performs a physical exam to determine the source of pain. Educators can employ a similar method as they determine the cause of the incorrect responses given by a student. The diagnosis may consume large amounts of time or be rapidly identified based on student work.

Prescription

Once a learning need is **D**iagnosed/identified, renaming in the case of our example, the teacher can then determine what **P**rescriptive action should be taken. In the medical profession, the instructor or doctor has multiple medicines or treatments that can be prescribed. These multiple medicines affect individuals in different ways based on body chemistry and make up. This is also true with education in relation to learning styles. In education, teachers should have multiple activities, learning situations, or practice methods that can be prescribed to help students understand. In our example the teacher could prescribe numerous interventions to help our student understand the renaming concept. (e.g., place value practice, peer discussion groups focused on a single problem, one-on-one discussion about place value, manipulative extensions, etc.)

As teachers formalize the work that is done in a classroom they will be able to define the learning that occurs in a classroom and what learning should take place in the future. There can be a fine line between instruction and assessment when educators use quality formative assessment tasks to guide instruction and learning (Leahy, et al., 2005). The E-D-P Model encourages teachers to evaluate student work, diagnose learning needs, and determine the best prescription for continued growth in knowledge. Some teachers complete these three stages daily in classrooms around the nation without defining the process. This model provides educators a method to formalize current practice and aid them in the implementation process.

Citations

Leahy, S., Lyon, C., Thompson, M., Wiliam, D. (November 2005). Classroom Assessment: Minute by Minute, Day by Day. *Educational Leadership*, 63:3, p.18-24.

McNamee, G.D., Chen, J.Q. (November 2005). Dissolving the Line Between Assessment and Teaching. *Educational Leadership, 63:3, p.72-76.*

Medical Metaphor T-Chart	
Physician	Educator
Why would a physician complete an Evaluation?	Why would an educator complete an Evaluation?
What would a physician use to make make a medical diagnosis?	What would an educator use to make a learning diagnosis?
When evaluation and diagnosis are complete what kind of prescription would be given?	When evaluation and diagnosis are complete what kind of prescription would be given?

E-D-P Assessment Form

Evaluation: ____________________

Name ____________________

Date ____________________

Task/Objective ____________________

( ) Individual ( ) Partner ( ) Group

Diagnosis:		
	Strengths	Weakness
1)		
2)		
3)		
4)		
5)		
6)		

Prescription:

E-D-P Assessment Form

Evaluation: ____________________

Name ____________________

Date ____________________

Task/Objective ____________________

( ) Individual ( ) Partner ( ) Group

Diagnosis:		
	Strengths	Weakness
1)		
2)		
3)		
4)		
5)		
6)		

Prescription:

E-D-P Assessment Form

Evaluation: ____________________

Name ____________________

Date ____________________

Task/Objective ____________________

( ) Individual ( ) Partner ( ) Group

Diagnosis:		
	Strengths	Weakness
1)		
2)		
3)		
4)		
5)		
6)		

Prescription:

E-D-P Assessment Form

Evaluation: ____________________

Name ____________________

Date ____________________

Task/Objective ____________________

( ) Individual ( ) Partner ( ) Group

Diagnosis:		
	Strengths	Weakness
1)		
2)		
3)		
4)		
5)		
6)		

Prescription:

E-D-P Assessment Form

Evaluation: _____

Students:	Diagnosis:							Prescription:				
Task:	Communication	Representation	Computation					Task #4	Comp. #6	Assignment #1		
1) Kyler	√-	√	√					X				
2) Jose	√	√+	√-							X		
3) Kyler	√+	√+	√+						X			
4) Sammy	√	√	√-							X		
5) Shelby	√-	√-	√-							X		

E-D-P Assessment Form	
Diagnosis:	Prescription:

*Copy to a label and place on student work.

E-D-P Assessment Form

Evaluation: _____

Students:	Diagnosis:							Prescription:				
Task:	Communication	Representation	Computation					Task #4	Comp. #6	Assignment #1		
1) Kyler	√-	√	√					X				
2) Jose	√	√+	√-							X		
3) Kyler	√+	√+	√+						X			
4) Sammy	√	√	√-							X		
5) Shelby	√-	√-	√-							X		

E-D-P Assessment Form	
Diagnosis:	Prescription:

*Copy to a label and place on student work.

Core Standard & Objective:	Assessment FOR Learning (Evaluate)			Assessment of Needs (Diagnosis)								
	Type of Assessment			Needs Missing Foundational Skills			Needs Basic Grade Level Core Skills			Needs Extension of Core Skills		
	Pre/Post Dates: /			Plan for Instruction (Prescription) Examples: Expicit Instruction using Graphic Organizers, Skill Specific Activities, Guided Practice, etc.								
Student Name	Limited Knowledge	Partial Knowledge	Mastery									

Mathematical Proficiency
Elementary CORE Academy 2007

How do educators know when a student "Gets It?" Elementary teachers interact with students daily using a variety of individual views regarding mathematical understanding. Success in mathematics is created through a student's composite view and aptitude in five areas of mathematics. In the book, *Helping Children Learn Mathematics,* we are introduced to this composite view of mathematics learning. The term mathematical proficiency is used to describe what it means when a person successfully learns mathematics.

Mathematical proficiency includes five strands:

1) **Understanding:** Comprehending mathematical concepts, operations and relations-knowing what mathematical symbols, diagrams, and procedures mean.
2) **Computing:** Carrying out mathematical procedures, such as adding, subtracting, multiplying, and dividing numbers flexibly, accurately, efficiently, and appropriately.
3) **Applying:** Being able to formulate problems mathematically and to devise strategies for solving them using concepts and procedures appropriately.
4) **Reasoning:** Using logic to explain and justify a solution to a problem or to extend from something known to something not yet known.
5) **Engaging: Seeing mathematics as sensible, useful, and doable-if you work at it-and being willing to do the work.**

It is critical to understand that each of these strands is interwoven and interdependent. Various views of success in mathematics emphasize one aspect of mathematical proficiency with the expectation that the other areas of mathematical knowledge will follow. Success in mathematics comes through achieving mathematical proficiency, which includes each of the five strands.

We see parents, students, and educators focus on only one strand of proficiency, which results in memorized facts that do not necessarily lead to mathematical success. This narrow treatment of math does not provide the strong basis of mathematical learning that students need.

As students learn all the aspects of mathematical proficiency, learning will become stronger, more durable, more adaptable, more useful, and more relevant. It is difficult to master any one of these strands in isolation and is therefore essential to teach the strands in an interconnected method. Developing the strands together builds a student's knowledge of any one strand through connected knowledge points that are memorable.

Citation

National Research Council. (2002). Helping Children Learn Mathematics. Mathematics Learning Study Committee, J. Kilpatrick and J. Swafford, Editors. Center for Education, Division of Behavioral and Social Sciences and Education. Washington, D.C.: National Academy Press.

Building Academic Vocabulary
Elementary CORE Academy 2007

Teaching students vocabulary that will be encountered during the study of content provides a solid background for a positive interaction with that content. Building academic vocabulary is much more than simply placing words upon a word wall or providing a matching exercise with a definition and new terms.

Initially the selection of the terms to be provided to students takes effort and time. Educators should identify key words that are important to the understanding of specific content areas, and are included in the Core Curriculum. The background work of identifying the terms is critical to providing an accurate direction for the subsequent instruction. However, the key to the success of building academic vocabulary ultimately rests upon the quality of the instruction provided by the teacher. Marzano and Pickering provide the following six-step Process for teaching new terms.

The Six-Step Process for Teaching Academic Vocabulary:

1) **Provide a description, explanation, or example of the new term.**
2) **Ask students to restate the description, explanation, or example in their own words.**
3) **Ask students to construct a picture, symbol, or graphic representing the term or phrase**.
4) **Engage students periodically in activities that help them add to their knowledge of the terms in their notebooks**.
5) **Periodically ask students to discuss the terms with one another.**
6) **Involve students periodically in games that allow them to play with the terms.**

With guidance and monitoring students have the ability to generate their own description and representations of vocabulary terms provided. The ownership of this process is valuable in that students see the term as a new tool that aids their learning. An integral step in the process of learning new vocabulary is the student notebook. As students add new terms to their notebook they also refine and update descriptions, which deepens and clarifies their understanding of the content and the terms.

Creating a deeper understanding of vocabulary terms will provide students with multiple points of learning as they encounter new content. These points of learning will broaden the knowledge base and allow students to develop an awareness of the language of learning.

Citation

Marzano, R.J., Pickering, D.J., (2005). *Building Academic Vocabulary Teachers's Manual* ASCD, Alexandria, VA.

Math I-2

Activities

Compare Numbers

How Low Can You Go?

Standard I:

Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 2:

Explain relationships and equivalencies among integers, fractions, decimals, and percents.

Intended Learning Outcomes:

2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.

Content Connections:

Math I-3; common denominator circles

Math Standard I

Objective 2

Connections

Background Information

Before teaching this lesson the students need to have a knowledge of the parts of a fraction (numerator and denominator) and prime and composite numbers. They also need to know that fractions with like denominators can be added and subtracted. Students will also need to understand equivalent fractions.

In this activity students will use fraction circles to build equivalent fractions and manipulate them to discover the least common denominator. Graphic organizers will be used to compare the circle fraction with the numerals involved.

Research Basis

Moore , D.W., (1984). A quantitative and qualitative review of graphic organizer research. *Journal of educational research*. 78, 11-17.

Two research reviews sought to sort out the accumulated evidence of graphic organizer effects on learning. Moore applied meta-analysis procedures to integrate research findings from 16 graphic organizer studies. Strong effects were obtained when students constructed graphic organizers after encountering content.

Loewenberg, D.B. (1992). Magical hopes manipulatives and the reform of math education. *American educator*. Summer, 14-18, 46-47.

This article focuses on manipulatives and how they effect students understanding. It also explains how manipulatives have changed education in a positive way. It discusses real-life examples of how manipulatives have enhanced student understanding.

Invitation to Learn

Materials

- ❑ *Picture Pie*
- ❑ Four inch circles
- ❑ *Fraction Cards*

In this activity the students will be given five four-inch circles that have been cut with a die cut machine. Show the students the book Picture Pie, by Ed Emberley. Ask the students if they recognize any fractions in the pictures. The students will then make a picture by cutting their circles into halves, quarters, and eighths. They will then use these pieces to create a picture (do not use thirds, sixths, or fifths). The teacher will then hand out a fraction card to each of the students. This card will tell the student how many pieces of their circles they can use to make their picture. After the students have had time to make their pictures and have labeled each fraction piece, they will be paired up and they will need to make up an addition mathematical equation using their pictures. The students will write this equation somewhere on their pictures. The pictures will be used for the next activity.

Instructional Procedures

Materials

- ❑ *Factor Tree*
- ❑ Factor Tree (overhead)
- ❑ *How to Find LCD*

1. Distribute two *Factor Tree* handouts per student.
2. Identify the parts of their equations (whole number, numerator, denominator, symbols used).
3. Have each student identify the denominator of their fraction on their picture they created in the previous activity by placing a finger on it.
4. Have the students look at their partner's denominators to see if they have found it.
5. Ask if their denominators are the same? Or different?
6. If there are some that are the same ask the students if the can be added? YES!
7. Most of the student's fractions should have different denominators. These denominators will be placed into the seed on the Factor Tree handouts. The students will then put their denominator into the denominator seed circle on one of their *Factor Tree* handouts.
8. The students will then put their partners denominator on the other *Factor Tree* handout that was given to them.
9. Explain that you will now be splitting the denominator seeds and watching them grow into prime factor fruit. (The teacher can tell the students that farmers pick their fruit when it is prime).
10. Show the students how this is done on the *Factor Tree* transparency.

11. After the students have come up with their factor fruit they will be using the *How to Find LCD* handout to accomplish the next part of the lesson. This handout should be placed into the student's math binder so it can be referred to in the future.
12. Count the number of times each prime number appears in each of the factorizations.
13. For each prime number, take the largest of these counts.
14. Write down that prime number as many times as you counted for it in step #2.
15. The least common denominator is the product of all the prime numbers written down.

Assessment Suggestions

- Observation
- E.D.P. (Evaluate, Diagnosis, Prescribe)
- Journal entry explaining how to find LCD

Curriculum Extensions/Adaptations/ Integration

- Advanced Learners could be given three or more denominators and asked to find the LCD.
- Special needs students could be given a multiplication array to help with facts they do not know.

Family Connections

- Students can go home and show their parents how to get factor fruit from a denominator seed using the *Factor Tree* handout.
- Students can create a story using the vocabulary from the lesson (denominator, numerator, whole number, prime number, etc.).

Additional Resources

Books

Picture Pie, by Ed Emberley; ISBN 0-316-78982-8

Web sites

http://www.glc.k12.ga.us/seqlps/sudisplay.asp?SUID=164

Fraction Cards

2 and 1/4	2 and 1/2	2 and 3/4	2 and 3/8	2 and 5/8
2 and 6/8	2 and 7/8	3 and 1/4	3 and 1/2	3 and 3/4
4 and 1/4	4 and 1/2	4 and 3/4	4 and 3/8	4 and 7/8
3 and 7/8	3 and 3/8	3 and 5/8	3 and 6/8	1 and 7/8

Fraction Cards

2 and 1/4	2 and 1/2	2 and 3/4	2 and 3/8	2 and 5/8
2 and 6/8	2 and 7/8	3 and 1/4	3 and 1/2	3 and 3/4
4 and 1/4	4 and 1/2	4 and 3/4	4 and 3/8	4 and 7/8
3 and 7/8	3 and 3/8	3 and 5/8	3 and 6/8	1 and 7/8

How to find LCD

To find the least common denominator using this method, **factor each of the denominators** into **primes**. Then for each **different** prime number in all of the factorizations, do the following...

1. **Count** the number of times each prime number appears in each of the factorizations.
2. For each prime number, take the **largest** of these counts.
3. Write down that prime number **as many times as you counted** for it in step #2.
4. The least common denominator is the **product** of all the prime numbers written down.

Example: We'll use the same fractions as above: **1/5**, **1/6** and **1/15.**

- **Factor into primes**
  - Prime factorization of **5** is **5** (**5** is a prime number)
  - Prime factorization of **6** is **2 x 3**
  - Prime factorization of **15** is **3 x 5**

Notice that the **different primes** are 2, 3 and 5.

- Now, we do **Step #1 – Count** the number of times **each** prime number appears in **each** of the factorizations...
  - The count of primes in **5** is **one 5**
  - The count of primes in **6** is **one 2** and **one 3**
  - The count of primes in **15** is **one 3** and **one 5**
- **Step #2** - For **each** prime number, take the **largest** of these counts. So we have...
  - The largest count of **2s** is **one**
  - The largest count of **3s** is **one**
  - The largest count of **5s** is **one**
- **Step #3 –** Since we now know the count of each prime number, you simply write down that prime number as **many times as you counted** for it in step #2.

Here are the numbers...

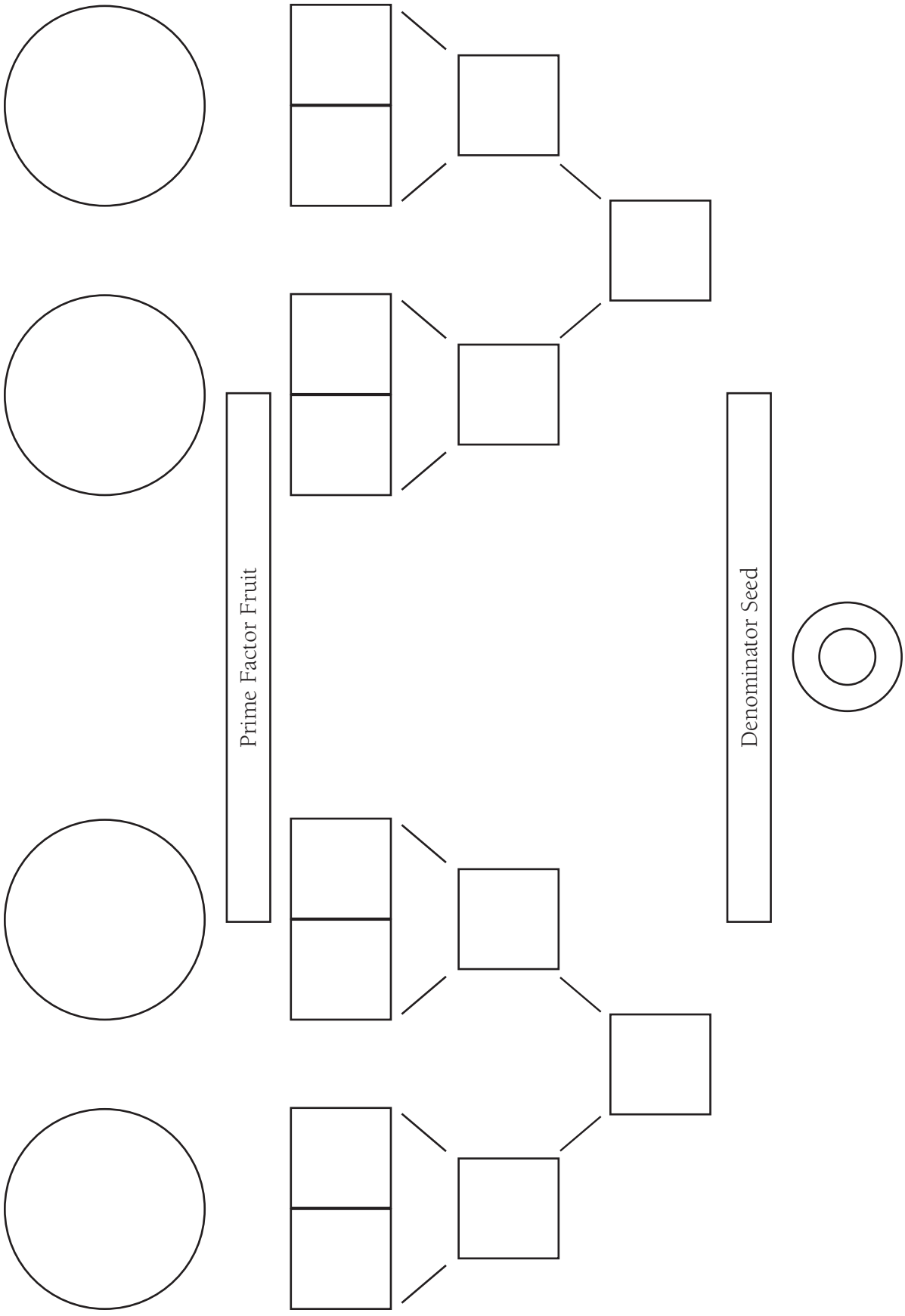
2, 3, 5

- **Step #4 –** The least common denominator is the **product** of all the prime numbers written down.

2 x 3 x 5 = 30

Therefore, the least common denominator of **1/5**, **1/6** and **1/15** is **30**.

Factor Tree



There's Nothing Improper About Them

Math Standard I

Objective 2

Connections

Standard I:

Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 2:

Explain relationships and equivalencies among integers, fractions, decimals, and percents.

Intended Learning Outcomes:

2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.

Content Connections:

Math I-3; Improper Fraction Bingo

Background Information

Before teaching this lesson, mixed and improper fractions should be introduced. The students need to know the definition to numerator, denominator, and whole number.

In this activity students will review what an improper fraction and a mixed number are and how they relate to one another using shapes. Students will then practice converting improper fractions to mixed numbers and vice versa using the well-known game Bingo.

Research Basis

Moore, D.W., (1984). A quantitative and qualitative review of graphic organizer research. *Journal of educational research. 78*, 11-17.

Two research reviews sought to sort out the accumulated evidence of graphic organizer effects on learning. Moore applied meta-analysis procedures to integrate research findings from 16 graphic organizer studies. Strong effects were obtained when students constructed graphic organizers after encountering content.

Ellis, E., (2004). Q&A: *What's the big deal with graphic organizers?* Retrieved December 30, 2006, from http://graphicorganizers.com/about.html.

This article answers some common questions about graphic organizers. It discusses many misconceptions about the use of graphic organizers and describes when and how to use them properly. It also discusses how graphic organizers can be valuable assessment tools.

Materials

- ❑ *Shape Shift Sheet*
- ❑ Assorted Shape Tiles.

Invitation to Learn

This activity is called shape shifting. In this activity the class is split up into pairs. Each pair is randomly given small shapes that can be manipulated and combined to make larger shapes (hand out enough for the pair to be able to make at least two big shapes). Explain that the triangles will need to be combined with the other triangles, squares with the squares, rectangles with rectangles. Do not combine different shapes. The teacher will then ask the question how many small shapes did it take to make your larger shape? Ask each pair to write down their mixed number and draw a picture on the *Shape Shift Sheet*. The groups that do not have whole shapes will need to write them down as fractions. Discuss how one small shape is a fraction of the larger shape.

Materials

- ❏ *Assorted shape tiles*
- ❏ *Improper Instructions #1*
- ❏ *Improper Instructions #2*
- ❏ *Blank Fraction Bingo Card*
- ❏ *Improper Fraction Answers*
- ❏ *Mixed Number Squares*
- ❏ Paper bag
- ❏ Bag of candy
- ❏ Computer

Instructional Procedures

1. Hand out *Improper Instructions #1* and *Improper Instructions #2*, have students use shapes to work through the instructions with the teacher as a review.
2. Pass out *Blank Fraction Bingo Cards* to students.
3. Have students fill in bingo cards with fractions located on overhead *Improper Fraction Answers*.
4. The teacher should have cut up *Mixed Number Squares*.
5. Students will take turns choosing the mixed number out of the bag.
6. Students will place a candy on the block on their card that has the appropriate improper fraction.
7. Students will take turns until a student covers a line (horizontally, vertically, or diagonally). The teacher may provide prizes (optional).
8. Students will use website to check their problems. Students should type in the improper fraction; the web site will change the fraction into a mixed number.

Assessment Suggestions

- Graphic Organizer (Bingo card)
- E.D.P. (Evaluate, Diagnosis, Prescribe)
- Observation of correctly converting fractions

Curriculum Extensions/Adaptations/ Integration

- Advanced learners could write story problems using improper fractions, orlist examples of real life situations where they observe improper fractions/mixed numbers.
- Learners with special needs could draw or use manipulatives to show examples of improper fractions/mixed numbers.

Family Connections

- Students could list examples of improper fractions/mixed numbers that they see at their home (improper fraction hunt).
- Students could ask parents if they use improper fraction/mixed numbers, then write a paragraph explaining how, when, and where their parents use them.

Additional Resources

Web sites

http://www.GraphiOrganizers.com

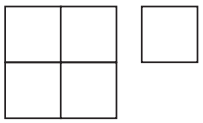
Shape Shift Sheet

Write the mixed number on the line	Write the mixed number on the line
_____	
_____	
_____	

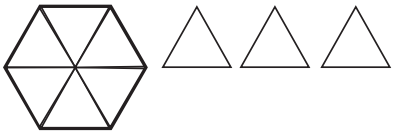
Improper Instructions #1

Improper Fractions are just fractions that have a larger numerator than denminator.
Like this one: **5/4**

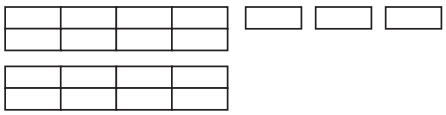
Use your unit blocks to make a model. **5/4 = 1 1/4**
Remember, **5/4** is the same as **1** whole with **1** extra.



Try another one **9/6**
Use your triangle blocks to make a hexagon.
Remember, **9/6** is the same as **1** whole with **3** extra.



What about this? **19/8**
19/8 is the same as **2 3/8**



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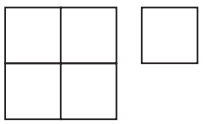

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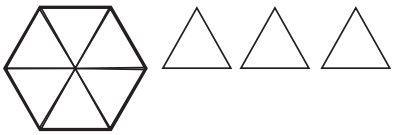

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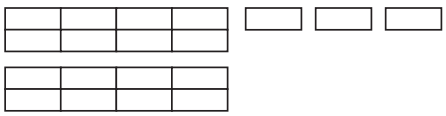
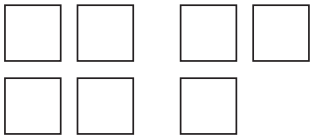
19/8 is the same as **2 3/8**



Improper Instructions #2

Improper Fractions are just fractions that have a larger numerator than denminator.
How about this **1 3/4**

Use your unit blocks to make a model.
Count the number of squares. **7 squares = 7/4**
Remember **7/4** is the same as **1** whole with **3** extra

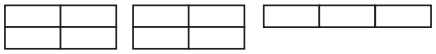


Can you change without the models?

2 3/4

Multiply these. Then add this to the product.
2 x 4 = 8 8 + 3 = 11 ⟵ This is your new numerator
The denominator stays the same. **11/4**

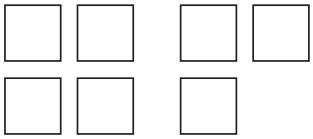
Check with your blocks. **2 3/4 = 11/4**



Improper Instructions #2

Improper Fractions are just fractions that have a larger numerator than denminator.
How about this **1 3/4**

Use your unit blocks to make a model.
Count the number of squares. **7 squares = 7/4**
Remember **7/4** is the same as **1** whole with **3** extra



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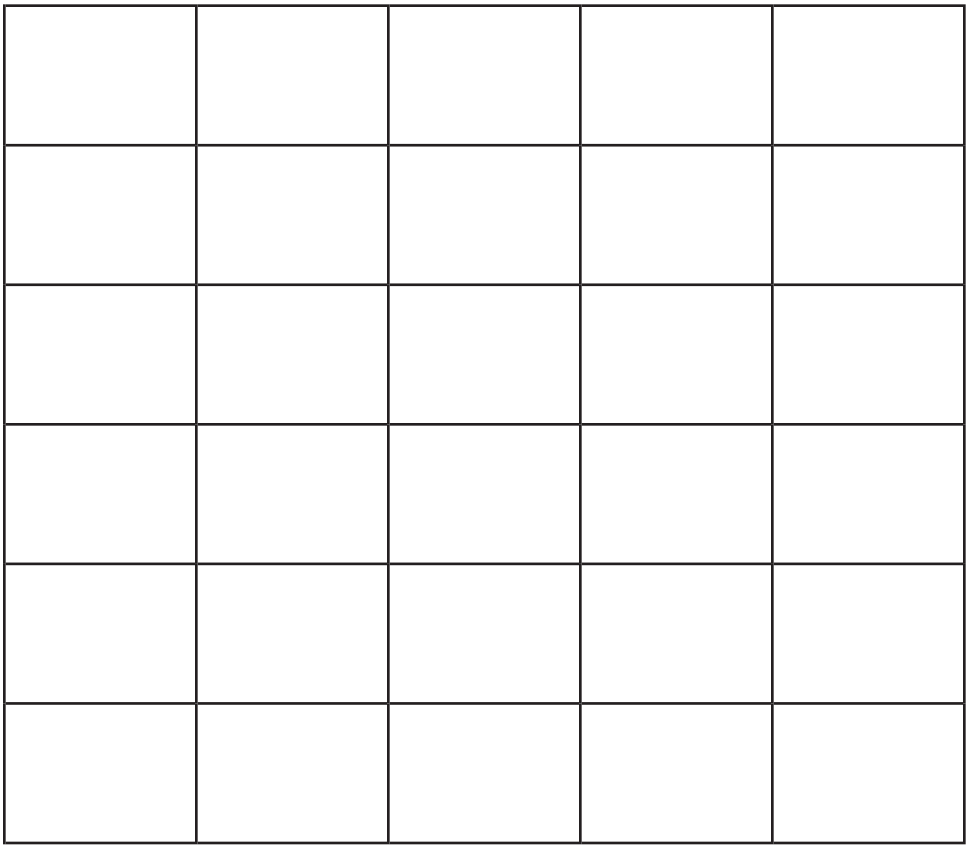
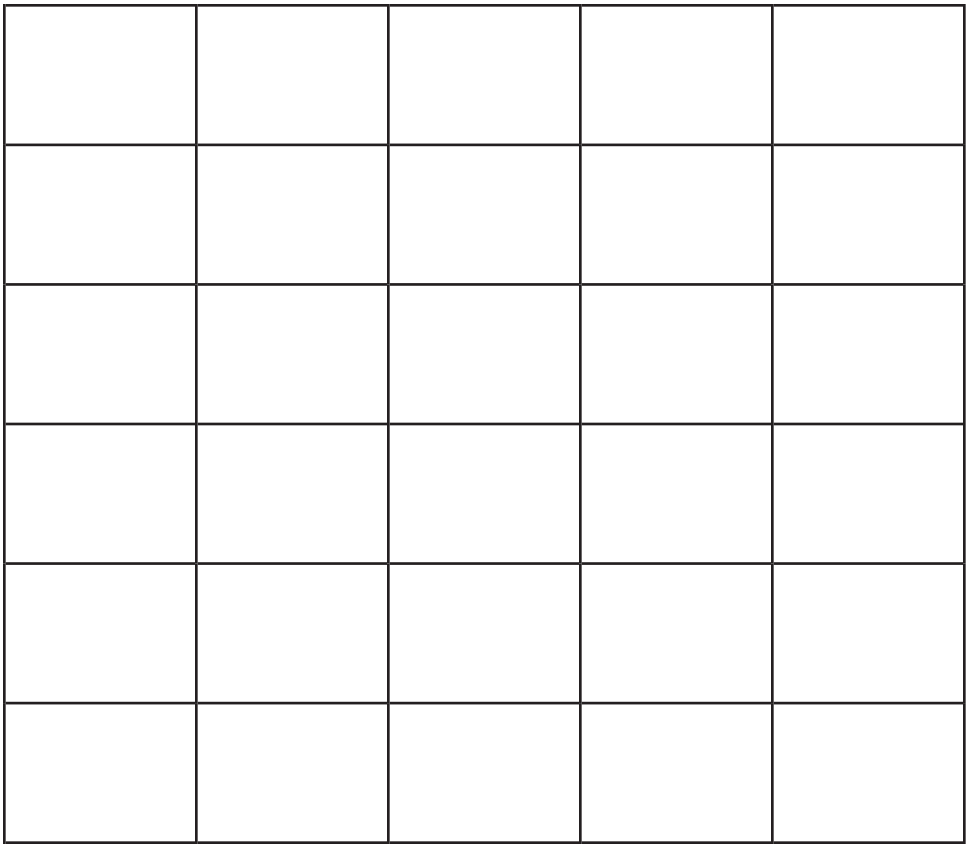
Fraction Bingo



Fraction Bingo



Improper Fraction Answers

Make transparency or copy to board.

$\frac{17}{6}$	$\frac{12}{7}$	$\frac{19}{4}$	$\frac{10}{3}$	$\frac{16}{6}$	$\frac{11}{4}$	$\frac{21}{7}$	$\frac{18}{5}$
$\frac{10}{6}$	$\frac{15}{3}$	$\frac{14}{3}$	$\frac{15}{2}$	$\frac{12}{7}$	$\frac{23}{6}$	$\frac{8}{6}$	$\frac{4}{3}$
$\frac{25}{4}$	$\frac{11}{6}$	$\frac{22}{4}$	$\frac{36}{7}$	$\frac{54}{7}$	$\frac{31}{9}$	$\frac{37}{6}$	$\frac{29}{7}$
$\frac{41}{8}$	$\frac{6}{6}$	$\frac{34}{5}$	$\frac{26}{8}$	$\frac{13}{5}$	$\frac{25}{4}$	$\frac{8}{3}$	$\frac{51}{8}$
$\frac{17}{4}$	$\frac{12}{7}$	$\frac{43}{5}$	$\frac{33}{6}$	$\frac{44}{9}$	$\frac{19}{4}$	$\frac{58}{9}$	$\frac{61}{6}$
$\frac{39}{9}$	$\frac{15}{7}$	$\frac{59}{7}$	$\frac{52}{5}$	$\frac{71}{8}$	$\frac{53}{4}$	$\frac{51}{6}$	$\frac{32}{9}$
$\frac{48}{6}$	$\frac{21}{6}$	$\frac{15}{4}$	$\frac{25}{3}$	$\frac{32}{7}$	$\frac{14}{6}$	$\frac{17}{6}$	$\frac{13}{6}$

Mixed Number Squares

Cut squares for bag.

$2\frac{5}{6}$	$1\frac{5}{7}$	$4\frac{3}{4}$	$3\frac{1}{3}$	$2\frac{4}{6}$	$2\frac{3}{4}$	3	$3\frac{3}{5}$
$1\frac{4}{6}$	5	$4\frac{2}{3}$	$7\frac{1}{2}$	$1\frac{5}{7}$	$3\frac{5}{6}$	$1\frac{2}{6}$	$1\frac{1}{3}$
$6\frac{1}{4}$	$1\frac{5}{6}$	$5\frac{2}{4}$	$5\frac{1}{7}$	$7\frac{5}{7}$	$3\frac{4}{9}$	$6\frac{1}{6}$	$4\frac{1}{7}$
$5\frac{1}{8}$	1	$6\frac{4}{5}$	$3\frac{2}{8}$	$2\frac{3}{5}$	$6\frac{1}{4}$	$2\frac{2}{3}$	$6\frac{3}{8}$
$4\frac{1}{4}$	$1\frac{5}{7}$	$8\frac{3}{5}$	$5\frac{3}{6}$	$4\frac{8}{9}$	$4\frac{3}{4}$	$6\frac{4}{9}$	$10\frac{1}{6}$
$4\frac{3}{9}$	$2\frac{1}{7}$	$8\frac{3}{7}$	$10\frac{2}{5}$	$8\frac{7}{8}$	$13\frac{1}{4}$	$8\frac{3}{6}$	$3\frac{5}{9}$
8	$3\frac{3}{6}$	$3\frac{3}{4}$	$8\frac{1}{3}$	$4\frac{4}{7}$	$2\frac{2}{6}$	$2\frac{5}{6}$	$2\frac{1}{6}$

Name _____ Date _____

Fraction Quiz

Change the improper fractions to mixed numbers.

$\frac{17}{3}$ = _____ $\frac{31}{6}$ = _____ $\frac{7}{2}$ = _____

$\frac{26}{4}$ = _____ $\frac{42}{8}$ = _____ $\frac{51}{9}$ = _____

$\frac{19}{5}$ = _____ $\frac{16}{3}$ = _____ $\frac{68}{7}$ = _____

Name _____ Date _____

Fraction Quiz

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Math Standard I-3&4

Activities

Modeling Operations

Playing with Remainders

Standard I:

Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 4:

Model and illustrate meanings multiplication and division.

Intended Learning Outcomes:

2. Become effective problem solvers by selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches to solve problems.
3. Reason logically, using inductive and deductive strategies and justify conclusions.
5. Connect mathematical ideas within mathematics, to other disciplines, and to everyday experiences.

Content Connections:

Language Arts Standard 1; oral language
Language Arts Standard 2; fluency

Math Standard I

Objective 4

Connections

Background Information

It often takes a leap of understanding for students to apply the procedural algorithm of division with remainders to real-world situations where remainders are encountered. A child who can easily calculate 40 divided by 6 = 6R4 will too often state 6R4 as the answer to the number of cars necessary to transport 40 children to a baseball game if 6 children can fit in each car. The activities in this section will first review the concept of division as proportional reasoning involving equal shares and then they will lead children to discover the three usual ways of dealing with remainders in real life: they are either used to round up to the next whole number, they are dropped and discarded, or they are split evenly among the participants.

Before beginning this lesson, students must be able to express remainders as fractions and decimals.

Materials

- ❑ 12 manipulatives
- ❑ Math journals

Research Basis

Wiebe, A., (1989). Proportionality: A major concept in mathematics—part II: Remainders—what are we to do with them? ***Aims newsletter, volume iii***, No. 7, 6-7.

Dr. Wiebe explores the gap between abstract answers to division problems with remainders and real-life situations where students encounter remainders. Expressing remainders as fractions and decimals are explored and applied, and the choices of rounding up, dropping, and sharing remainders are introduced.

Martinez, J.G.R., (2000). Look smart. *Early years,* January 2000. Retrieved January 12, 2007 from http://www.findarticles.com.

Engaging children in math story problems is easier when the stories have real plots and good endings. By engaging students in the plot, they become interested in solving the math situations, rather than routinely solving a page of "word problems." Additionally, the enthusiasm generated motivates students to write their own stories, developing new problems within the story context, and acting out the story line.

Invitation to Learn

Distribute a set of 12 counting objects to each child. (They may be cubes, blocks, chips, etc.) Tell the students that they each have a set of 12 objects. Then ask the students to divide their sets into four fair shares. Guide them to create four sets with three objects in each set. Discuss the term "fair shares" if it is not part of your usual vocabulary. It means every set has the same number of objects, the dividend is divided equally by the divisor. Then write the following equation on the board and ask the children to copy it into their math journals.

$$4\overline{)12}\ 3$$

Ask what is different from the usual way of writing a division problem. They should notice that the number 1 is written above the divisor. What is significant about the number 1? Take several ideas from students. Lead them to discover that the 1 is implied in every division problem, because the quotient is how many items are in 1 fair share. Then have the children write the following two statements in their journals:

- 1 fair share contains 3 objects.
- 4 fair shares contain 12 objects.

Explore with the children the relationships between the numbers as they discover the proportions: 1/4 = 3/12; 1/3=4/12 and 1X12=3x4. Write all the true statements on the board and have the children list them in their journals.

Next, copy these three equations on the board:

$$4\overline{)12}\ ?\qquad ?\overline{)12}\ 3\qquad 4\overline{)?}\ 3$$

Ask what is the question in the first equation. (The students are asked to form 4 fair shares from 12 objects.) What is the question in the second equation? (The students are asked to find how many fair shares of 3 each can be made with 12 objects.) What is the question in

the third equation? (Students are asked to find how many objects must be used to make 4 fair shares containing 3 objects each. This case involves multiplication rather than division.) Have the children build each situation with their manipulatives, knowing that even though the problem looks the same each time, in the first instance the question is the number of fair shares in each set. In the second instance, the question is the number of sets, and in the third instance, the question is the total number of objects.

Introducing division as proportional reasoning prepares children for equivalency in fractions; proportionality in ratios, proportions and percents; and provides a more concrete understanding of division as the process of creating fair shares.

Instructional Procedures

1. Divide the class into three groups. Each script has enough parts for eight actors. Additional class members could be used to direct, create, and manage props, etc. If you have a really large class, you may wish to double one or more of the scripts and perform the same play(s) twice. The plays also work well in a readers' theater format, shortening preparation time.
2. Practice the plays: "*Round-up!,*" "*Sharing is Very Important!*" and "*You Just Drop It!*"
3. Present each play to the whole class. During and after the presentations, the class completes the graphic organizer *Playing with Remainders.*
4. As a class, discuss the different applications of remainders in the three plays using the graphic organizer to illustrate the different ways each play uses remainders in real life. You may wish to have the children trim the edges of the graphic organizer and glue it into their journals as a reference.
5. As a whole class or in partners complete the worksheet *Remainder Stories.*
6. Note: Another option for these plays is to use them as center activities, with each child participating in each play, using no audience but discussing each play separately as a whole class. This option may increase student engagement.

Materials

- ❏ Math journals
- ❏ *Round-up!*
- ❏ *You Just Drop It!*
- ❏ *Sharing is Very Important!*
- ❏ *Playing with Remainders*
- ❏ *Remainder Stories*
- ❏ Paper
- ❏ Scissors
- ❏ Tape
- ❏ Markers

Assessment Suggestions

- Formative assessment: Check for accuracy as students complete their graphic organizers, participate in the discussion following the presentation of the plays, and solve the word problem worksheet.
- Final assessment: In a word-problem test, students should be able to supply the correct answer and explain in words what they did with their remainders (dropped, shared, or rounded up).

Curriculum Extensions/Adaptations/ Integration

- After reading or acting out these plays, children could write their own stories or plays where the characters must interpret remainders correctly in real-life situations.
- Children new to the United States could be encouraged to set new plays in their homeland countries with names, food, and problem-solving situations common to their life experiences.

Family Connections

- Assign students to create two to five word problems at home using members of their families and either real or made up situations that require the correct use of remainders.
- Which use of a remainder is most common? Give students a few days to collect data at home about which scenario is most common—dropping, rounding or sharing. They might be allowed situations on TV in addition to real-life occurrences. After collecting data, a bar graph could be constructed comparing the three types of remainders' frequency.

Additional Resources

Books

Teaching with the Brain in Mind, by Eric Jensen; ISBN 1-4166-0030-2

Web site

http://www.edhelper.com

"Round-up!"

Scene 1

Narrator: "Our play begins in the family room of a modern home where two 11-year-old children are gathered around the TV."
Mom: (entering from the kitchen) "Tyler! Nikki! I want to talk to you!" (The children stay glued to the TV.) "Nikki! Tyler! You'll want to hear what I have to say!"
Tyler and Nikki together: "Okay, Mom, what's up?"
Mom: "We have our plans for the family reunion. We're going to a dude ranch with all the cousins. You'll spend a week away from the TV—riding horses, rafting a river. You might even get to take part in a cattle round up!"
Tyler: "Cool! When are we going?"
Mom: "Next Friday."
Nikki: "Can I ride with my cousin Brittany?"
Mom: "Everyone's coming to our house to meet. I don't think we'll need to take everyone's cars. Gas is so expensive, we might as well take as few cars as possible."

Scene 2

Narrator: "Now the setting changes to the front yard outside Nikki and Tyler's house. All the relatives are gathered to go to the reunion together. Nikki is standing by her mom, not really listening. Tyler is standing next to his dad."
Tyler's dad: "All right, everyone! Stand together! How many people do we have? Let's see…I think we have 23 people, counting all the children. Each car we are taking has 5 seat belts, so how many cars do we need?"
Tyler: "That's easy, Dad. 23 divided by 5 = 4R3. We need 4R3 cars!"
Narrator: "A strange voice is heard above the crowd. Everyone freezes as it calls in a low, slow, Western drawl…"
Voice: "Round-up!"
Narrator: "Slowly the action returns, but Tyler acts as if he has been struck by lightning."
Tyler: "Dad, no. We don't need 4R3 cars. I have to round up that remainder. We need 5 cars."

Scene 3

Narrator: "We join our cast outside the main lodge at the *No-Remainder Ranch*. Nikki and her mom are standing in front of Wrangler John."
Wrangler John: "Welcome, everyone! Gather round so I can assign you a bunk. Let's see, there are 23 of you, and I can put 4 in a cabin. How many cabins do I need?"
Nikki: "I can do that problem in my head! 23 divided by 4 = 5R3. We need 5R3 cabins!"
Narrator: "Again, a strange voice is heard above the crowd. Everyone freezes as it calls in a low, slow, Western drawl…"
Voice: "Round-up!"
Narrator: "Slowly the action returns, but now Nikki acts as if she has been struck by lightning."
Nikki: "Wait! 5R3 cabins doesn't make any sense. I need to round up the remainder! We need 6 cabins for 23 people. One bunk will just have to be empty."

Scene 4:
Narrator: "Join Nikki and Tyler's family in a clearing next to the bank of a fast-moving river. Family members are putting on life jackets and waiting for instructions from Wrangler John. Nikki and her cousin Brittany are standing together."
Wrangler John: "Be sure your life jacket is on properly. The river is fast and you will encounter some class 4 rapids. We haven't lost anyone yet this year, and we don't expect to. Each raft holds 6 guests, plus a guide who knows the river well. Let's see…we have 23 guests. How many rafts do we need to take?"
Brittany: "That's easy! 23 divided by 6 = 3R5. We need 3R5 rafts!"
Narrator: "Everyone suddenly becomes silent. Tyler and Nikki look around, as if expecting the voice. And, sure enough, seemingly out of nowhere, it calls…"
Voice: "Round-up!"
Narrator: "Brittany rubs her forehead as if she has been hit by lightning. Then she excitedly calls…"
Brittany: "Wait! 3R5 rafts doesn't make any sense! I have to round up! We need 4 rafts."

Scene 5:
Narrator: "It's night time, and the guests of the ***No-Remainder Ranch*** are seated around a campfire. They are listening to Wrangler John tell stories about the mountains around them."
Wrangler John: "Do you want to hear another story?"
Tyler, Nikki, Brittany: "Yes!"
Wrangler John: "Well, okay. This story has been around for a long time, and folks around here believe it to be true. Have you wondered, since you have been here, how the ranch got its name?" (The assembled guests nod their heads, and Wrangler John continues.) "A long time ago, people around here couldn't stay safely through the winter. Gathering enough provisions took too much work, and it made sense to go down to town where it was warmer. So, early every November, after all the harvestin' was done, the animals were driven down to lower ground. There wasn't much of a ranch here, and it wasn't named at all. Those who worked here came back after the animals were secure and gathered the last of their things and then went back to town in their wagons. This happened year after year without incident. That is until 1906. In 1906 snow came earlier than usual, and the cattle drive had to be put together quickly. Five men came back to the ranch after that, just to tidy things up and get the last of their provisions. They had to hurry, because a fierce storm was just a few hours away, and getting stuck at the ranch over the winter would be no picnic."
Narrator: "Wrangler John looked carefully over his audience to see that they were paying attention. No one spoke."
Wrangler John: "The men had one wagon, drawn by two work horses. They divided into teams of two for the last of their chores and then got into the wagon and drove away. What they didn't realize was that in dividing 5 men by 2, they had left one man out. 'Scorch', as thy called him, because he usually burned dinner, had no partner, no job, and had been left to winter alone at the ranch. By the time they realized they'd forgotten 'Scorch', the high country was buried in three feet of snow and it was too late to go back and search for him."
Tyler: "Was he ever seen again? What happened to him?"
Wrangler John: "No. The next spring, when the wranglers returned to the ranch, a careful search was conducted. But no remains were ever discovered. However, a strange legend surrounding

'Scorch's disappearance is told today. It is said that he protects people all over these parts from being left behind. Whenever a group is dividing into sets, and an important remainder might be forgotten, he calls in a low, slow drawl, 'Round-up!' and the group remembers to include the remainder. In fact, it's after one such experience that the name of the ranch was changed to the *No-remainder Ranch*. But it's just a legend. I don't know anyone personally who has heard the voice…"

Narrator: "The crowd grows silent as Brittany, Nikki, and Tyler look at each other in amazement. They know THEY'VE heard the voice. Each time they were about to leave an important remainder behind, the voice instructed them to round up. And, as if to remind them forever, once more they heard the low, slow, Western drawl…"

Voice: "Round-up!"

"Sharing is Very Important"

Scene 1

Narrator 1: "Have you ever had a little brother or sister turn into a nosy tattle-tale? Scott and Travis did. Their little sister Samantha turned five and thought she was the boss of everything! But one day, they decided they were glad to have her around. That day, Little Samantha saved Travis' life."

Narrator 2: "Our story begins in the Hunter family's back yard. Scott is 13, Travis is 10, and Samantha is 5. As usual, Scott and Travis are trying to accomplish something, and Samantha is in their way."

Scott: "Travis, hand me that rope. I want to tie knots in it. We can climb up it to get into our tree fort." (Travis hands Scott the rope.) "If we cut it in two pieces, we can use ½ for the front door and ½ for the back door. Hmm…we have 11 feet of rope. How long does each piece need to be?"

Travis: "That's easy! 11 divided by 2 = 5R1. Each piece needs to be 5R1 feet long."

Samantha: "I'm telling Mom! You're not sharing!"

Travis: "Not sharing what? What are you talking about?"

Scott: "Just ignore her. You said what about the rope?"

Travis: "Each piece needs to be 5R1 feet long."

Narrator 1: "Samantha didn't want to be ignored, so she went to their mother for help. Soon Mom came into the back yard."

Mom: "Boys, Samantha says you aren't sharing. Don't you know that you need to share whenever you can? Sharing is very important."

Narrator 2: "With that bit of advice, Mom went back into the house. And Travis and Scott went back to work."

Travis: (with a long look at Samantha) "Scott, something Mom just said made sense. We can share this remainder. Each piece of rope can be 5 ½ feet long. Thanks, Samantha. You actually helped us with this tree house."

Scene 2

Narrator 1: "Within a few days the tree house was finished, and it was time to have a sleepover in it. Scott and Travis decided there was room for 4 sleeping bags, so each of them invited his best friend. As soon as it was dark, they climbed the ropes and settled in."

Narrator 2: "Of course, no one really sleeps at a sleepover, right? Within minutes, on each boy's sleeping bag heaped a pile of treasure—whole bags of candy, stacks of baseball cards for trading, and Game Boy's and Ipods for later, when the talking wore thin."

Scott: "Justin, are you ready to share your Airheads? I want a blue one."

Justin: "There are 17 in the bag and we have 4 kids. How many does that give each of us?"

Travis: "I'm good at division. 17 divided by 4 = 4R1. Each kid gets 4R1 Airheads."

Narrator 1: "At just that moment, Samantha's head popped up in the entrance to the tree fort."

Samantha: "Hey, you guys forgot to invite me. And you're not sharing! Don't you know sharing is very important?"

Scott: (with a long look at Samantha) "Travis, you're right about Samantha. Sometimes she says just the right thing. We can share that remainder. Each kid gets 4 Airheads, and we can divide the last one into 4 pieces. We'll each get 4 1/4 Airheads."

Travis: "I'm okay about Airheads, but what I'm really eyeing is Hector's Reese's Peanut Butter Cups. Hector, how many Reese's do you have?"

Hector: "There are 10 in the bag. And I know that 10 divided by 4 = 2 R2, so we each get…"
Samantha: "If you don't start sharing, I'm telling Mom again!"
Travis: (looking at Samantha) "Okay, we'll share the remainder. 10 divided by 4= 2R2. But if we share the remainder, we'll each get 2 1/2 Reese's."
Scott: "Now, Samantha, get lost. This is a BOY tree house!"
Scene 3
Narrator 2: "The tree house was a big hit. For most of the summer Scott and Travis had a sleepover in it at least once a week. But in mid-August, Scott's friend, Justin, had another big idea."
Justin: "Hey guys, let's do a survival camp-out on Slickrock Mountain!"
Hector: "What's a survival camp-out?"
Justin: "It's when we each go our own way and we have to stay alone all night, without a tent or anything!"
Travis: "Is it safe?"
Scott: "Sure! We don't go very far from each other—just far enough to not see each other. We'll stay at the old mine camp."
Narrator 1: "The boys got permission from their parents, and decided to meet in exactly one week with all their camping gear. They would get ready in Scott and Travis' back yard."
Scene 4
Narrator 2: "It was still hot at 7 p.m. when the boys gathered for their campout. The mine camp was just a couple of miles from Scott and Travis' house, so they decided to hike in and then separate at bedtime. They piled their stuff on the concrete patio, just to be sure they had thought of everything."
Justin: "Does everyone have a flashlight?"
Boys: "Yeah!"
Hector: "What about mosquito repellent?"
Boys: "Yeah!"
Scott: "What about matches?"
Justin: "Oh, I don't."
Travis: "Neither do I."
Hector: "I don't either. Scott, do you?"
Scott: "Yeah, I have a few books of them. Do you guys want to use some?"
Boys: "Yeah!"
Scott: "Okay, I have 5 books. With 4 boys, we each get…"
Travis: "I know! 5 divided by 4 = 1R1. We each get 1R1 books of matches."
Narrator 1: "All of a sudden Samantha appeared around the corner of the house."
Samantha: "Hey, guys, are you sharing yet? If you don't share, I'm telling Mom. Sharing is very important!"
Travis: (looking at Samantha) "Hmm…can we share this remainder? I guess so. We'll split open the book and each take 5 matches. How's that for sharing?"
Scene 5
Narrator 1: "Travis, Scott, Justin and Hector took off for the old mine camp. There they cooked a fine dinner over a large campfire and then sat late into the evening, roasting marshmallows and counting the constellations. Travis absent-mindedly threw his book of matches into the fire and watched it flare up and then disappear."

Narrator 2: "Then it was time to find a solitary place to camp. The boys decided to each take 100 steps in a different direction, so they wouldn't be too far away. Travis chose to walk 100 steps up the side of Slickrock Mountain, hoping to find a sheltered niche against a fir tree."
Travis: "This is a good spot. I think I'll sleep here."
Narrator 1: "And so he fell asleep with a sweatshirt for a pillow and fir branches for a blanket. He slept soundly all night."
Narrator 2: "But when he woke up, he wasn't sure at all where he was. Everything looked different by daylight. He tried calling his brother and friends, but no one answered. Knowing the rules of survival, he didn't hike away—instead he waited in the same spot for someone to find him."
Travis: "I'll stay right here. I know my family will come looking soon."
Narrator 1: "The day passed without anyone finding Travis. And as night came, it looked like it would snow. Suddenly, Travis was afraid he was in real trouble!"
Travis: "I wish I hadn't thrown all my matches in the fire. I could really use a signal fire about now. I bet if I built a fire, my family would find me soon, and I would stay warm too."
Narrator 2: "Travis reached his hand deep into his left front pocket, wishing he had that book of matches. Almost unbelievingly, his hand found the five remainder matches that Samantha had insisted be shared."
Travis: "Hey! I have 5 matches! It is important to share a remainder! I can build a fire with these and my family will rescue me!"
Narrator 1: "It didn't take long for Travis to build a roaring fire, with smoke and flames reaching high into the sky. It didn't take much longer for Travis' family to find him, high on Slickrock Mountain, and to bring him home. They had been searching all day, but they had been on the opposite side of the mine camp."
Narrator 2: "Travis was very happy to be home. He'd survived all right, because of Samantha's insistence that they share a remainder."
Travis: "Thanks, Samantha."
Samantha: "Sharing is very important!"

"You Just Drop It!"

Scene 1

Narrator: "Marisol and Shailee have lived next door to each other for nine of their eleven years, and except for a few fights every now and then have been best friends the entire time. Marisol puts up with Shailee's moodiness, and Shailee puts up with Marisol's clumsiness. Best friends have to forgive each other—that's why they are best friends. They do have a lot in common: both love sports and good music, and right now both of them want to be veterinarians when they grow up. In fact, they are discussing their future right now."

Marisol: "Shailee, how do you think we are going to be able to afford all the school it takes to be veterinarians?"

Shailee: "I think we should start saving our money now!"

Marisol: "What money? I don't even get an allowance."

Shailee: "Well, let's start a business! If we can start earning money, we'll be able to start saving money."

Marisol: "What could we do? We're a little old to sell lemonade."

Shailee: "Actually, I've been thinking about this for a while. We could set up a roadside stand and sell baked goods, lemonade, and flowers. If we are smart about it, I think we could earn a lot of money."

Scene 2

Narrator: "Marisol's mom had a connection with a flower wholesaler, and Shailee's grandma made the best cookies and brownies in town. It didn't take long for Marisol and Shailee to have a whole kitchen full of flowers and goodies to sell."

Shailee: "Hold these flowers, Marisol, while I tie ribbons around them. I want to put them in bunches of 7. Hmm…we have 37 flowers. How many bunches of 7 can we make with 37 flowers?"

Marisol: "That's easy! We can make 5R2 bunches. Oops, Shailee, I'm sorry! I dropped those two flowers! I'm so clumsy! I accidentally dropped the remainder!"

Shailee: "Don't worry. We couldn't make a bunch with just two flowers; we couldn't use them anyway. It was okay to drop the remainder. We really only could make 5 bunches. Now, hand me those brownies…"

Narrator: "Marisol gave Shailee a tray of brownies."

Shailee: "Okay, we can fit 6 brownies on each plate. How many brownies do we have?"

Marisol: "We have 34 brownies. With 6 on a plate, we can fill 5R4 plates of brownies…Oh, no! I accidentally dropped 4 brownies! They're just crumbs on the floor now! I'm sorry I'm so clumsy."

Shailee: "Marisol, you are clumsy, but you dropped just the remainder, and we couldn't use it anyway. No one would want to pay for a plate that was only 2/3 full. We still have 5 plates of brownies. But will you carefully hand me the chocolate chip cookies? I need to count them."

Marisol: "I'll count them. There are 50 cookies, and they look really yummy! Let's put them in sets of 8. That way we'll have 6R2 plates."

Narrator: "Marisol started handing the tray of cookies to Shailee. But just before Shailee grasped them, Marisol slipped on the brownie crumbs on the floor and two cookies slid off."

Marisol: "Shailee, I just dropped two of the cookies! What will we do now?"

Shailee: "Marisol, don't worry about that! You just dropped the remainder! They were extra anyway. We still have 6 plates of cookies to sell."

Scene 3

Narrator: "Every weekend Shailee and Marisol sold flowers and baked goods at their roadside stand. Soon their business grew so large that they had to hire more employees."
David and Sean: "Marisol and Shailee, thanks for letting us work for you. What do you want us to do?"
Shailee: "David, will you put the flowers in bunches of 7 and tie ribbons around them? Try to choose colors that look good together."
Marisol: "Sean, will you put the brownies and cookies on plates and wrap them? We sell brownies in sets of 6 and cookies in sets of 8."
David: "There are 58 flowers. That means I can make 8 bunches of flowers with a remainder of two. What do I do with the remainder?"
Sean: "There are 40 brownies. That means I can make 6 plates with a remainder of two. And there are 46 cookies. I have enough cookies for 5 plates with a remainder of 6. What do I do with the remainder?"
Narrator: "Shailee and Marisol just looked at each other and laughed. Then they said to David and Sean…"
Marisol and Shailee: "YOU JUST DROP IT!"

Scene 4

Narrator: "The money kept piling up in the bank, and in a little over fourteen years Marisol and Shailee had their very own veterinary clinic. They were still best friends—Shailee was still moody and Marisol was still clumsy. On a June morning as Shailee was standing behind the front desk, she was surprised to see David and Sean show up at their shop. It had been years since the childhood friends had seen one another."
Shailee: "Hi, David and Sean. It's wonderful to see you! Marisol, (she calls into a room behind her), will you bring drinks for everyone?" (She turns back to David and Sean.)
David: "Hi! I'd like you to meet my wife, Brianna, and our Golden Retriever, Lucky. We brought him to you so he could get his shots. You remember Sean, don't you?"
Sean: "Hi! This is my Chihuahua, Bentley. He needs shots too."
Marisol: (entering with 6 cups of water) "Hi! Here, have some water. Oh, I brought 6 cups when I only needed five…"(she slips, spilling one of the cups onto the floor.) "Uh, oh—I dropped it! Oh, well—it was an unimportant remainder anyway. There's nothing wrong with dropping an unimportant remainder!"

Name _____ Date _____

Playing with Remainders

Listen to the play *"Round-up!"* and then answer the following four questions:

1. 23 people went to the family reunion. If 5 people could fit in each car, how many cars were needed to take people to the *No-Remainder Ranch*?

2. At the *No-Remainder Ranch* 4 people were assigned to each cabin. How many cabins were needed for the 23 people?

3. While they were at the ranch, the family members went on a river-rafting trip. If 6 people could fit on each raft, how many rafts were needed for the 23 people?

4. Why did Tyler and Nikki have to round up their remainders each time in this story?

Listen to the play *"You Just Drop It!"* and then answer the following four questions:

1. How many bunches of 7 flowers could Shailee and Marisol make with 37 flowers?

2. How many plates of 6 brownies each could be made with 34 brownies?

3. How many plates of 8 cookies each could be made with 50 cookies?

4. Why did Shailee and Marisol have to drop their remainders each time in this story?

Listen to the play "*Sharing is Very Important?* and then answer the following three questions:

1. Scott and Travis cut an 11-foot rope into two equal pieces. How long was each piece?

2. Four boys are sharing 17 Airheads equally. How many Airheads does each boy get to eat?

3. Why didn't the remainders in this story need to be rounded up or dropped?

Write a good rule for what to do with remainders.

when you need to round up the remainder	when you need to drop the remainder	when you need to share the remainder equally

Name _____ Date _____

Remainder Stories

Answer each question with a complete sentence. Then tell how you used the remainder (whether you rounded up, dropped, or shared the remainder equally). Last, tell why you used the remainder the way you did. Each problem is worth four points (1 point = correct answer; 1 point = complete sentence; 1 point = correct use of remainder; 1 point = explanation for use of remainder).

1. Skyler is helping his mother plan a wedding breakfast for his older sister, Jessica. They are expecting 63 family members to attend, and they are using round tables that seat 8 guests each. How many tables will be needed to seat 63 people?

2. Skyler and his sister, Rylie, are preparing flower bouquets as centerpieces for each table at the wedding breakfast. They hope to have enough to decorate the table that is displaying the wedding cake as well. They have 67 carnations and wish to put 6 carnations in each bouquet. How many bouquets can they make with 67 carnations?

3. Rylie is going to the zoo for her 12th birthday party, and she is taking 9 friends. The zoo has a new baby giraffe, and groups of 3 children are allowed at a time in a special viewing room to see the giraffe and his mother. How many tours will it take for Rylie and her 8 friends to see the giraffe?

4. At Rylie's 12th birthday party, she wants to give each of her friends a jar with a variety of candy from the candy store. She has 5 friends coming, and she has 113 individually wrapped pieces of candy. If she gives each person the same number of pieces of candy, how many pieces will each friend receive in her candy jar?

5. While Rylie is celebrating with her friends, Skyler's mom gave him $10.00 to share equally with his three best friends so they could buy candy too. How much money do Skyler and his friends each get to spend?

6. Jessica is making curtains for her new apartment. She has 15 yards of material to make 2 sets of curtains. How much fabric can she use for each set?

7. Jessica's mother is serving punch at the wedding reception. She has a punch bowl that holds 106 ounces of punch. How many 8-ounce servings can be poured from the punch bowl when it is full?

Divisibility Rules

Standard I:

Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 3:

Use number theory concepts to develop and use divisibility tests; classify whole numbers to 50 as prime, composite, or neither; and find common multiples and factors..

Intended Learning Outcomes:

1. Demonstrate a positive learning attitude toward mathematics.
3. Reason logically, using inductive and deductive strategies and justify conclusions.

Content Connections:

Science 1; Use science process and thinking skills

Math Standard I

Objective 3

Connections

Background Information

The rules of divisibility are simple formulas for understanding how fair shares can be created from large numbers without practicing long or short division. Students usually come to fifth grade with an implicit understanding about why numbers are divisible by 2, 5, and 10, but it is important in fifth grade to make that understanding explicit. Additionally, the formulas for dividing numbers by 3 and 9 must be taught, since they are rarely discovered by children. It is helpful to separate the formulas for 2, 5, and 10 (which depend on the digit in the ones column) from the formulas for 3 and 9 (which depend on the sum of the digits and the formula for 6, which combines the rules for 2 & 3). Note that there are simple formulas for divisibility by 4, and 8, (as well as more complicated formulas for larger numbers) but they are not part of the Utah fifth grade Core Curriculum requirements. Information about these formulas is included in the curriculum extensions section for interested students.

This lesson should be sequenced after division with whole numbers has been reviewed and practiced, division with remainders has been reviewed and practiced, and students are familiar with vocabulary terms dividend, divisor, and quotient. It may also be used to review prime and composite, since every number greater than 2 that is divisible by 2, 3, 5, 6, 9, or 10 is composite; also, when discussing divisibility, students will probably remember that all numbers are divisible by one and themselves.

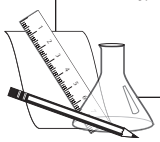
Research Basis

Furner, J.M., Yahya, N., Duffy, M.L. (2005). Teach mathematics: Strategies to reach all students. *Intervention in school and clinic, Vol. 41, No. 1, 16-23.*

In 2000 the National Council of Teachers of Mathematics identified "equity" as the first principle for school mathematics, meaning all children have the right to understand mathematical principles. This article offers 20 teaching strategies to reach the wide variety of learning styles and ability levels in our classrooms as we aim to meet the equity principle. Good lessons may incorporate several of these 20 strategies at one time: we may draw, explain verbally, organize conceptually, demonstrate manipulatively, and practice kinesthetically. Grouping heterogeneously and connecting culturally helps our lessons cross learning barriers and provide opportunities for children to help each other learn.

Materials

- ❑ Index Cards
- ❑ *Divisibility Test*
- ❑ Calculators
- ❑ *Divisibility Rules*
- ❑ Chart paper
- ❑ Markers



Ball, D., (1992). Magical Hopes: Manipulatives and the reform of math education: *American educator,* Summer 1992.

Although this article is 15 years old, its concerns are still valid: are we using manipulatives wisely when we teach mathematics to children? What are the relative merits of different concrete objects? Are lessons using manipulatives sensible to adults because we already understand the concepts they are designed to represent? As teachers it is important for us to understand the purpose behind the manipulatives we use when we design instruction, and it is vital for us to link the activities using manipulatives to the mathematical concepts explicitly for children to make important connections.

Invitation to Learn

Divide the class into teams of three members each. One member is the director, one the recorder, and one the materials coordinator. Each team takes four index cards and writes a different digit from 0-9 on each card. Then, from the four choices of digits, the team makes a list of all the possible four-digit number combinations using each digit once. There will 24 possible number combinations. Next, have the students each take a graphic organizer, *Divisibility Test,* with columns for the numbers they created, plus the columns for 2, 3, 5, 6, 9, and 10 listed across the top. Using calculators if you wish, have the students divide each of their 24 numbers by 2, 3, 5, 6, 9, and 10 to decide if their numbers divide evenly without leaving remainders. If the number divides evenly, have the students write "yes" in the column on the graphic organizer. If the number does not divide evenly, have the students write "no" in the column on the graphic organizer.

After the graphic organizer is complete, have each team record their "yes" examples on chart paper hanging around the room, one piece for

each of the numbers 2, 3, 5, 6, 9, and 10. Once this is done, have each team make a hypothesis about a "rule" for divisibility for each of the numbers 2, 3, 5, 9, and 10. Have them record their hypotheses on the graphic organizer labeled *Divisibility Rules*. It is important that each child have his or her own copy of the two graphic organizers because the next part of the lesson is done as a whole class.

Instructional Procedures

1. After teams have completed their *Divisibility Test* graphic organizer, recorded their numbers on the chart paper, and made hypotheses about divisibility on their *Divisibility Rules* graphic organizer, have them return to their individual seats for a whole-class lesson.
2. Using the chart paper lists as summaries of numbers generated by the class teams, discuss each chart and have the students share their hypotheses of divisibility rules. Guide their discussions to the correct rules for each number, and have them write them on the graphic organizer. Then have them trim the edges of their graphic organizers and glue them into their math journals for later referencing.
3. Ask the students if it is possible to divide their rules into two main categories, using a Venn Diagram to compare and contrast the categories. Lead them to separate the numbers where the ones digit determines the divisibility (2, 5, 10) from the numbers that require adding all the digits (3, 9). Have them complete a Venn Diagram in their math journals while you model one on the board.
4. Play *Divisibility Rocks* using students' journals as reminders of the divisibility rules. Note: if this game is used as one station in a variety of center activities, fewer sets of the game will need to be produced.

How to play *Divisibility Rocks*:

1. Divide the class into groups of two to six students per game. (An ideal size game is three students because each player will always have a job.)
2. Give each group one game set. Each set requires a deck of cards, a bag of rocks, and a *Divisibility Key*.
3. Divide the cards face down evenly among members of a group. Discard any remaining cards. Pile the rocks in the center of the game.

Materials

- ❏ Math journals
- ❏ *Divisibility Rocks Cards*
- ❏ *Divisibility Pebbles*
- ❏ *Divisibility Key*

4. Decide which person will be the first Player. The person to his or her right will hold the *Divisibility Key* and the person to his left will be the Challenger.
5. The first Player turns over his or her top card. The person holding the *Divisibility Key* asks, "is it divisible by 2?" If the Player answers, "yes," then he takes a rock from the pile. The process is repeated with the numbers 3, 5, 6, 9, and 10, with the Player taking a rock for each "yes" answer. (An example is a student would receive three rocks for the number 10 because it is divisible by 2, 5, and 10.)
6. Then the person with the *Divisibility Key* turns to the Challenger and asks, "do you want to challenge him?" If the Challenger believes any answers were incorrect, he or she may answer "yes," telling what numbers are believed to be incorrect.
7. If the Challenger is correct, he gets all the rocks from the Player. If the Challenger is incorrect, he forfeits the next turn.
8. If the Player is wrong and the Challenger refuses to challenge, the person with the *Divisibility Key* corrects the turn and corrects the number of rocks taken.
9. The play then moves clockwise to the left, with the past Player now responsible for the *Divisibility Key*, and the Challenger becoming the next Player.
10. At the end of a round, the person with the most rocks collects the cards used in the round and all the rocks are returned to the center of the game. A new round is played.
11. At the end of a round, if there is a tie, both Players involved in the tie turn over their next card and collect the rocks for that card. Whoever holds the card that earns the most rocks wins the round.
12. A player is out when he is out of cards; the Player with all the cards at the end of the game is the winner.
13. To shorten the game, the teacher may set a time limit; the person with the most cards at the end of the allocated time is the winner.

Assessment Suggestions

- Pre-assessment: Observe the children's hypotheses as they write on their graphic organizers to see if their prior knowledge about divisibility is accurate, especially with numbers 2, 5, and 10.

- Formative assessment: Check for accuracy as students write correct rules on their graphic organizers, complete their Venn diagrams, and verbalize their responses during the Divisibility Rocks game.
- Final assessment: Using the Divisibility Test graphic organizer as a master, list ten numbers with a variety of divisibilities and have the students complete the chart with "yes" or "no" answers.

Curriculum Extensions/Adaptations/ Integration

- Advanced learners may enjoy discovering the rule of divisibility for 4 (last two digits are either 00 or are divisible by 4), and the rule for 8 (last three digits are divisible by 8). Rules for higher numbers are available on the web sites listed in the additional resources.
- Why do the rules work? Advanced learners may enjoy hypothesizing about the rules for 3 and 9—why are adding digits meaningful? Explanations are given on the web sites listed in the additional resources.
- Heterogeneous grouping for the invitation to learn and the card game help struggling learners through cooperative processes.
- The scientific method is used to discover mathematical absolutes. Children may recognize science vocabulary as the rules for divisibility are discovered through the formation of hypotheses, the gathering of data, the formation of conclusions, etc. Explicit teaching of these vocabulary terms strengthens both areas of science and mathematics.

Family Connections

- Can the rules of divisibility apply to real-life situations? Ask the students to find at least one example after school where the rules of divisibility shorten the task of creating equal shares. An example: mom fries scones and makes 15 scones. She knows they can be divided evenly among the five people in her family. Repeat this assignment for a few days, until everyone has had a chance to discover an example.
- Are the rules of divisibility for 3 and 9 unfamiliar enough to mystify people? How many people can you surprise by asking them to tell you a 10-digit number and then you telling them

whether it is divisible by 3 and 9? Record their numbers and their comments and report back to class for a discussion.

Additional Resources

Web sites

http://www.homeschoolmath.net/teaching/md/division_rules.php

http://www.math.about.com/library/bldivide.htm

http://argyll.epsb.ca/jreed/math7/strand1/1104.htm

http://www.mathforum.org/k12/mathtips/ward.html

Name _____ Date _____

Divisibility Test

1. Write the 24 numbers you created in the first column.
2. Decide if your numbers are divisible by 2, 3, 5, 6, 9, or 10. Write yes or no in the correct colums.

Number	2	3	5	6	9	10
1.						
2.						
3.						
4.						
5.						
6.						
7.						
8.						
9.						
10.						
11.						
12.						
13.						
14.						
15.						
16.						
17.						
18.						
19.						
20.						
21.						
22.						
23.						
24.						

Name _____ Date _____

Divisibility Rules

Number Divisible By	My Hypothesis	The Actual Rule
2		
3		
5		
6		
9		
10		

How to Play "Divisibility Rocks"

1. Take a deck of cards, a Divisibility Key, and a bag of rocks.
2. Divide the cards face down evenly among players. Discard any extras.
3. Place the pile of rocks in the center of the playing circle.
4. Decide who is first. The person to his right is in charge of the Divisibility Key.
5. The first player turns over his top card and decides if the number on the card is divisible by 2, 3, 5, 9, and 10. He takes one rock from the center pile for each "yes" answer.
6. If the player to the left disagrees, he or she may "challenge" by saying "Challenge!" Then both players appeal to the person holding the key to see who is right. If the challenger is correct, that person gets the rocks. If the challenger is incorrect, the original player gets to keep the rocks and the challenger loses his or her turn.
7. Play continues clockwise with each person taking a turn, rotating the person who holds the key and the person who is the challenger.
8. When every player has had a turn, the rocks are counted. Whoever has the most rocks gets to keep all the cards from that turn. The rocks are returned to the center pile.
9. If there is a tie, both players involved in the tie turn over their next card and collect the rocks for that card. Whoever holds the card that earns the most rocks wins the round.
10. A player is out when he or she is out of cards; the player with all the cards at the end of the game is the winner.
11. To shorten the game, the teacher may set a time limit; the person with the most cards at the end of the allocated time is the winner.

Divisibility Rocks Cards

24	34
35	36
44	46
48	55

Divisibility Rocks Cards

56	57
60	62
65	72
74	75

Divisibility Rocks Cards

80	84
98	115
117	128
130	140

Divisibility Rocks Cards

150	160
171	175
190	196
200	216

Divisibility Rocks Cards

240	256
260	285
308	309
335	338

Divisibility Rocks Cards

385	408
429	438
447	495
524	567

Divisibility Rocks Cards

625	657
666	669
700	711
715	728

Divisibility Rocks Cards

735	741
770	771
849	888
915	960

Divisibility Rocks Cards

1115	1135
1280	1324
2204	2220
2225	2318

Divisibility Key

Number	2	3	5	6	9	10
24	Y	Y	N	Y	N	N
34	Y	N	N	N	N	N
35	Y	Y	N	Y	Y	N
36	Y	Y	N	Y	Y	N
44	Y	Y	N	N	N	N
46	Y	N	N	N	N	N
48	Y	Y	N	6	N	N
55	N	N	Y	N	N	N
56	Y	N	Y	N	N	N
57	N	Y	N	N	N	N
60	Y	Y	Y	6	N	Y
62	Y	N	N	N	N	N
65	N	N	Y	N	N	N
72	Y	Y	N	Y	Y	N
74	Y	N	N	N	N	N
75	N	Y	Y	N	N	N
80	Y	N	Y	N	N	Y
84	Y	Y	N	Y	N	N
98	Y	N	N	N	N	N
115	N	N	Y	N	N	N
117	N	Y	N	N	Y	N
128	Y	N	N	N	N	N
130	Y	N	Y	N	N	Y
140	Y	N	Y	N	N	Y
150	Y	Y	Y	Y	N	Y
160	Y	N	Y	N	N	Y
171	N	Y	N	N	Y	N
175	N	N	Y	N	N	N
190	Y	N	Y	N	N	Y
196	Y	N	N	N	N	N
200	Y	N	Y	N	N	Y
216	Y	Y	N	Y	Y	N
240	Y	Y	Y	Y	N	Y

256	Y	N	N	N	N	N
260	Y	N	Y	N	N	Y
285	N	Y	Y	N	N	N
308	Y	N	N	N	N	N
309	N	Y	N	N	N	N
335	N	N	Y	N	N	N
338	Y	N	N	N	N	N
385	N	N	Y	N	N	N
408	Y	Y	N	Y	N	N
429	N	Y	N	N	N	N
495	N	Y	Y	N	N	N
524	Y	N	N	N	N	N
567	N	N	N	N	N	N
625	N	N	Y	N	N	N
657	N	Y	N	N	Y	N
666	Y	Y	N	Y	N	N
669	Y	Y	N	Y	N	N
700	Y	N	Y	N	N	Y
711	N	Y	N	N	Y	N
715	N	N	Y	N	N	N
728	Y	N	N	N	N	N
735	N	Y	Y	N	N	N
741	N	Y	N	N	N	N
770	Y	N	Y	N	N	Y
771	N	Y	N	N	N	N
849	N	Y	N	N	N	N
888	Y	Y	N	Y	N	N
915	N	Y	Y	N	N	N
960	Y	Y	Y	Y	N	Y
1115	N	N	Y	N	N	N
1135	N	N	Y	N	N	N
1280	Y	N	Y	N	N	Y
1324	Y	N	N	N	N	N
2204	Y	N	N	N	N	N
2220	Y	Y	Y	Y	N	Y

Math I-2

Activities

Fractions

Equal Shares

Standard I:

Students will expand number sense to include integers and perform operations with whole numbers, simple fractions, and decimals.

Objective 2:

Explain relationships and equivalencies among integers, fractions, decimals, and percents.

Intended Learning Outcomes:

3. Reason logically using inductive and deductive strategies and justify conclusions.
4. Communicate mathematical ideas and arguments coherently to peers, teachers, and others using the precise language and notation of mathematics.

Content Connections:

Math II-1; recognize, analyze, and use patterns
Math IV-1a; using customary units of measurement

Math Standard I

Objective 2

Connections

Background Information

This exploration is best done following a class discussion lead by the teacher of what a fraction is and what it really represents. Often time students are intimidated with the concept of fractions. Have them relax and just think of the fraction as another way to write or express a division equation. Mathematicians are known to be very "efficient" folks and seem to always find the most efficient way to write, express, and communicate things quickly. They are always anxious to move on and get the job done. Show the students the ÷ symbol. Do you see the fraction model in this symbol? The line means to "share equally." The denominator is the number of shares. Students relate to sharing with friends, so you might refer to the denominator as, "how many friends you will be sharing with?" The numerator is the portion of the shares to be considered. You can actually cover that number of shares with your hand to give the students an action cue to depend on. The following exploration and experience with the manipulatives is to as much uncover what the students know as much as to allow them to discover!

Research Basis

Zull, J.E. (2004). The art of changing the brain. *Educational leadership*. September 2004

This article explores the fact that learning should feel good. When a student is experiencing, exploring, developing connections, and learning then positive emotions are generated. This biochemical reward of learning is not provided by explanations from the teacher,

Materials

- ❑ Licorice rope

but by the student developing their own idea and ownership of those ideas. It goes on to discuss that the way we feel always influences our brain and strengthens growth and wiring. The article shares some best practices for teachers to optimize learning in the classroom.

De Geest, E., & Watson, A., (2004). Instilling Thinking. *Mathematics Teaching*. June 2004.

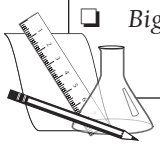
This article shares research done to identify and develop ways of stimulate mathematical thinking. It explores the common practice of giving students in the lowest achieving group repetitive, simplified mathematics. When studies show that more good is done helping learners develop thinking skills and understanding throughout every level of mathematics lessons. This with a teachers high expectations help a student's self-awareness that they are learning and progressing. Students showed significant gains in self-esteem and their ability and willingness to engage with extended, unfamiliar, and complex tasks.

Invitation to Learn

Provide each group of four to five students with a single licorice rope. Ask them to share this one licorice rope with the group "equally." Don't allow them to eat the shares until you have a chance to talk as a class. This activity will only take a few minutes. Children share everyday, all day long, so they will jump right in and get busy sharing. Travel among the groups and listen for snippets or phrases being said during the sharing. Pull the class together and share things you heard and go right into a discussion of "sharing equally." Depending on the responses and your assessment of understanding you might need to "share" more objects on the overhead with the class. Then share the traditional fraction model. Discussing and clarifying as needed. Let them eat!

Materials

- ❑ Manipulative set (one per class)
- ❑ Construction paper-variety of colors
- ❑ Scissors
- ❑ *Can You Make?*
- ❑ *Share Equally*
- ❑ *If This Is...?*
- ❑ *Matching Bars Game*
- ❑ *Big Inch*



Instructional Procedures

1. Provide single manipulative sets on a table or area where students have access: fraction circles, fraction pieces, pattern blocks, fraction bars, 12-centimeter cubes, yard stick, ruler, egg carton, *Cake* worksheet, number line 0-1, and *Clock* worksheet. These are suggestions only. You can pare down the choices or add others depending on the degree of challenge you wish to deal with and availability.
2. Challenge the students to show, model, and name as many equal shares of the tool, object, or manipulative being used.

3. Invite the individual groups to pick the manipulative of their choice.
4. Circulate among the groups and assess knowledge level, vocabulary being used, and progress. Allow about ten minutes for group members to interact on the task.
5. Then suggest to the class the use of a graphic organizer, *Can You Make?*, to help record findings.
6. Some explanation of how the *Can You Make?* graphic organizer is set up and its use may be needed and this usually works itself out if you take a manipulative and start working through an example on the overhead.
7. Students continue working and complete organizer to twelfths. There is value in the sketching of the manipulative pieces and a few groups may be confronted with having to construct sevenths, ninths, and elevenths. Provide construction paper of colors not represented in manipulative pieces.
8. Groups will then present findings to the total class. This will give an opportunity for you to discuss proper vocabulary in depth and clear up misconceptions that might have come up. This is a rich exploration. Students access prior knowledge, organize findings, organize patterns, interpret patterns, identify equivalents, process proportions, use estimation, order relationships of fractions to the whole, and make connections to other concepts in mathematics.
9. Have groups record the patterns that developed as they filled in the graphic organizer. Do this in traditional fraction representation.
10. Discuss and write equivalent fractions on a chart, overhead or chalkboard as they are shared.
11. In their math journal or on the bottom of the *Can You Make?* graphic organizer have them write: What I learned or discovered from this experience?

Assessment Suggestions

- A performance assessment is built into the completion of *Can You Make?* graphic organizer.
- Observation and interview of the experience.
- Journal writing of students reflection on the experience.

Curriculum Extensions/Adaptations/ Integration

- An extension for advanced learners would be the worksheet, *Share Equally* and/or *If This Is...?*
- Adaptations for learners with special needs or as a re-teaching activity for a smaller group is the *Matching Bars Game*.
  1. Place the fraction bar set of 16 pieces face down in the center of the group. Arrange them in equal rows and columns.
  2. To determine which player goes first: each player picks one of the face down bars. The player with the greatest amount shaded goes first. Replace the bars face down.
  3. Now take turns turning over two bars per turn that have the same shaded amount. If the shaded amounts are the same, he keeps the bars and goes again.
  4. If the two bars do not have the same amount shaded, they are turned over again and the next student takes a turn.
  5. Play continues until all the bars have been matched. The student with the most matching bars wins.
- Another adaptation for those needing further practice in linear and length models is the folding activity *Big Inch*.
  1. Pretend that the paper is going to be an inch magnified.
  2. Fold the paper in half end to end.
  3. How many sections do you have?
  4. Draw a line along the fold about three inches long.
  5. Write ½ under that line.
  6. Now fold the paper in half again.
  7. How many sections do you have?
  8. Draw a shorter line on each fold.
  9. Write 1/4 under the first line, 2/4 on the second line, and 3/4 on the last fold line that was created.
  10. Now fold the paper in half again.
  11. How many sections do you have now?
  12. Fill in the numbers on the folds created.
  13. Now fold the paper in half again.
  14. How many sections?
  15. Fill in the numbers on the folds created.
  16. Discuss the experience and allow students to measure with their Big Inch.